MATH 132 SOLUTION FOR FINAL EXAM FALL 2009

1) Evaluate $\int_0^{\frac{1}{2}} e^{sin(x)} \cdot \cos(x)\, dx$.

$A)\, 0\ \ B)\, 1\ \ C)\, e\ \ {}^*D)\, e-1\ \ E)\, e^2-1\ \ F)\, \frac{e^2-1}{2}\ \ G)\, e^{\frac{1}{2}-1}\ \ H)\, e^1-1\ \ I)\, e^2-1$

Solutions: $(u = sin(x)\, du = cos(x)\, dx) = \int_0^1 e^u du = e^u\big|_0^1 = e-1\ \ (D)$

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2) Evaluate $\int_0^1 x \sin(2x)\, dx$.

$A)\, 0\ \ B)\, \frac{\sqrt{2}}{2}\ \ C)\, 1-1\ \ {}^*D)\, \frac{-1}{2}\ \ E)\, 21\ \ F)\, \frac{\sqrt{3}}{2}-1\ \ G)\, -21\ \ H)\, 1\ \ I)\, \frac{1}{4}\ \ J)\, 21-1$

Solutions: $u = x,\ dv = sin(2x)dx\ ,\ du = dx,\ v = -\frac{1}{2}cos(2x)\,dx$.

$\int x \sin(2x)\, dx = -\frac{1}{2}x cos(2x) + \int \frac{cos(2x)}{2} dx = -\frac{1}{2}x cos(2x) + \frac{sin(2x)}{4}$.

$So\ \int_0^1 x \sin(2x)\, dx = -\frac{1}{2}x cos(2x) + \frac{sin(2x)}{4}\Big|_0^1 = \frac{-1}{2}\ \ (D)$

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3) Use partial fractions to evaluate $\int_1^2 \frac{x+4}{x(x+1)}\, dx$.

$A)$ 1.24 $B)$ 1.38 $C)$ 1.46 *D) 1.56 E) 1.69 F) 1.75 G) 1.80 H) 1.91 I) 2.14

Solutions: $\frac{x+4}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{(A+B)x+A}{x(x+1)} \Rightarrow A = 4\ and\ B = -3.$

$Then\ \int_1^2 \frac{x+4}{x(x+1)}\, dx = \int_1^2 \frac{4}{x} - \frac{3}{x+1}\, dx = 4ln(x) - 3ln(x+1)\big|_1^2 =$

$4ln(2) - 3ln(3) + 3ln(2) = 7ln(2) - 3ln(3) = 1.556 \sim 1.56\ \ (D)$

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4) Find what becomes of the integral $\int \frac{x^3}{4+x^2}\, dx$, when you make the substitution x = 2 tan()) , $-\frac{1}{2} <) < \frac{1}{2}$.

${}^*A)\, 4\int tan^3())d)\ \ B)\, \frac{1}{4}\int \sec^3())d)\ \ C)\, \frac{1}{2}\int \sec(2))d)\ \ D)\, 4\int \cot^2())d)$
$E)\, \frac{1}{2}\int \csc^2())d)\ \ F)\, \frac{1}{4}\int sec^3()) - 1\ d)\ \ G)\, 2\int tan^2())\, d)$
$H)\, \int 1 - tan^3())d)\ \ I)\, 2\int 1 + \cot^2())d)\ \ J)\, \int 1 + \csc^3())d)$

Solution: $Then$ dx=2 $\sec^2())d)$. $\int \frac{x^3}{4+x^2}\, dx$ œ $\int \frac{8\tan^3())}{4+4\tan^2())} 2\sec^2())d)$ œ
$4\int \frac{\tan^3())\sec^2())}{2\sec^2())} d) = 4\int tan^3())d)\ \ (A)$

5) Find the area of the region(s) enclosed by $f(x) = (x-1)^2$ *and* $f(x) = 1 - x$.

$A)\,\frac{1}{4}\;\; B)\,\frac{3}{4}\;\; C)\,1\;\; D)\,\frac{5}{4}\;\; E)\,2\;\; {}^*F)\,\frac{1}{6}\;\; G)\,\frac{5}{6}\;\; H)\,\frac{7}{6}\;\; I)\,3\;\; J)\,\frac{5}{3}$

Solutions: $(x-1)^2 = 1 - x$ *for* $x = 0$ *and* $x = 1$. *On* $[0,1]$, *line is above.*
So $A = \int_0^1 (1-x) - (x-1)^2\,dx = \int_0^1 x - x^2\,dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{6}\;(F)$

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6) Find the volume of the solid you get by **revolving** the region enclosed by the curve $x = \sqrt{y}$, the lines $x = 0$ and $y = 4$, **about the B-axis .**

A) 12.61 $B)$ 14.4.1 $C)$ 17.2 1 $D)$ 19.81 ${}^*E)$ 25.61 $F)$ 28.41
$G)$31. 41 $H)$ 33.81 $I)$ 35.61 $J)$ 38.21

Solution: Region of revolution in x-y plane is enclosed by $y = x^2$ *and* $y = 4$ *with* $0 \leq x \leq 2$. *By disc method* $V = 1\int_0^2 16 - x^4\,dx = 1(16x - \frac{x^5}{5})\Big|_0^2 = 25.61\,(E)$

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7) Suppose **S** is the solid whose base in the region bounded by the parabola $y = 4 - x^2$ and the line $y = 0$ and whose cross sections perpendicular to the y-axis are squares. For each y between 0 and 4 **find a formula** for the cross section **A(y)**.

A) y^2 B) $4\,y^2$ $C)\,4y$ $D)\,2\,y^2$ $E)\,8 - 2y$ FÑ $y^2 + 4$ $G)\,4y + 2$ $H)\,12 - y^2$
$I)\;2y + 4$ ${}^*J)\;16 - 4y$

Solution: *The region has its top at the vertex of* $y = 4 - x^2$, $(0,4)$, *and its bottom on the* $x-axis$, *so the interval is* $0 \leq y \leq 4$. *For each* y *the side of the square has length* $2x$, *where* x *is the* $x-coordinate$ *of a point* (x,y) *on* $y = 4 - x^2$.
Therefore $A(y) = 4x^2 = 4(4-y) = 16 - 4y\;\;(J)$

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8) Calculate the arc length of the curve $y = \frac{2}{3}x^{3/2}$, $0 \leq x \leq 3$.

A) $\frac{1}{2}\;\; B)\,\frac{5}{3}\;\; C)\,2\;\; D)\,\sqrt{5}\;\; E)\,\sqrt{2}\;\; F)\,\frac{7}{3}\;\; G)\,\frac{3}{2}\;\; H)\,\sqrt{\frac{1}{2}}\;\; {}^*I)\,\frac{14}{3}\;\; J)\,4$

Solution: $\frac{dy}{dx} = x^{\frac{1}{2}}$, $(\frac{dy}{dx})^2 = x$, *so length is* $l = \int_0^3 \sqrt{1+x}\,dx$
$(u = 1 + x,\, du = dx) = \int_1^4 u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}}\Big|_1^4 = \frac{2}{3}(8-1) = \frac{14}{3}\;\;(I)$

9) Determine whether the improper integral $\int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx$ converges and if so determine its value .

A) 0 B) 2 C) $\frac{4}{3}$ $D)\ 4$ $E)\ \sqrt[3]{7}$ $F)\ 6^{\frac{2}{3}}$ $^*G)\ 6$ $H)\ 8$ $I)\ \frac{1}{2}$ $J)\ diverges$

<u>Solution</u>: $\int (8-x)^{-1/3}\,dx = -\frac{3}{2}(8-x)^{2/3} \Rightarrow \int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx =$
$-\frac{3}{2}(8-x)^{2/3}\Big|_0^8 = 0-(-\frac{3}{2}\,8^{2/3}) = 6\ (G)$

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10) A force of 20 **N** is required to hold a spring stretched 5 **m** beyond its natural length. **How much work** is done in stretching it from 5 **m** to 10 **m** beyond its natural length ?

A) 50 J B) 90 J C) 110 J D) 130 J *E) 150 J F) 170 J G) 190 J H) 210 J I) 230 J J) 250 J

<u>Solution</u>: $Given\ 20 = 5k \Rightarrow k = 4\ and\ F(x) = 4x. Therfore\ W = \int_5^{10} 4x\,dx =$
$2x^2\Big|_5^{10} = 150\,J\ \ (E)$

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11) Find the solution to the initial value differential equation $\frac{dy}{dt} = -y^2$, $y(0) = \frac{1}{2}$.

$^*A)\ y = (t+2)^{-1}$ $B)\ y = t + \frac{1}{2}$ $C)\ y = (t+\sqrt{2})^{-2}$ $D)\ y = (t+\frac{1}{\sqrt{2}})^2$
$E)\ y = (t+1)^2 - \frac{1}{2}$ $F)\ y = (t+2)^{-1} - \frac{1}{2}$ $G)\ y = (t+2)^{-2} - \frac{1}{2}$
$H)\ y = e^{t-\ln(t)}$ $I)\ y = sin(2t) + \frac{1}{2}$ $J)\ y = cos(t) - \frac{1}{2}$

<u>Solution</u> : $\int -\frac{1}{y^2}dy = \int dt + C.\ Then\ \ \frac{1}{y} = t + C.\ \ y(0) = \frac{1}{2} \Rightarrow C = 2.$
$So\,we\,get\ \ y = (t+2)^{-1}\ (A)$

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12) Find the sum of the geometric series $3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \frac{48}{625} - \ldots\ldots$

$A)\ \frac{15}{4}$ $^*B)\ \frac{15}{7}$ $C)\ \frac{13}{5}$ $D)\ \frac{9}{5}$ $E)\ \frac{11}{25}$ $F)\ \frac{25}{12}$ $G)\ \frac{25}{3}$ $H)\ \frac{125}{27}$ $I)\ \frac{125}{3}$ $J)\ \frac{25}{17}$

<u>Solution</u>: $3 - \frac{6}{5} + \frac{12}{25} - \frac{24}{125} + \frac{48}{625} - \ldots\ldots = 3(1 - \frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \ldots).$
$We\,have\,geometric\ series\ with\ c = 3\ \ and\ \ r = -\frac{2}{5}. Therefore\ it\ converges$
to

$$\frac{3}{1+\frac{2}{5}} = \frac{15}{7}\quad (B)$$