Math 132 FINAL SPRING 2001

Solutions

1. A particle moves along the x–axis with velocity $v(t) = 1/\sqrt{t}$, $t > 0$. Find the total distance traveled by the particle on the interval $[1 \leq t \leq 4]$.

(A) 0 (B) 1/2 (C) 1 (D) 3/2 (E) 2

(F) 5/2 (G) 3 (H) 7/2 (I) 4 (J) 9/2

$$\text{Distance} = \int_1^4 t^{-1/2}\,dt = 2t^{1/2}\Big|_1^4 = 2\cdot 2 - 2\cdot 1 = 2$$

2. For what positive value of k does the function $y = \cos kt$ satisfy the differential equation

$$y'' + 16y = 0?$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(F) 6 (G) 7 (H) 8 (I) 9 (J) 10

$y = \cos kt$

$y' = -k \sin kt$

$y'' = -k^2 \cos kt$

$0 = y'' + 16y = -k^2 \cos kt + 16 \cos kt$

$\therefore \quad k^2 = 16$

$k = 4 \quad (\text{since } k > 0)$

3. Evaluate

$$\int_0^{\pi} x \sin 2x \, dx.$$

(A) 0 (B) $\sqrt{2}/2$ (C) $\pi - 1$ (D) $-\pi/2$ (E) 2π

(F) $(\sqrt{3}/2) - \pi$ (G) -2π (H) 1 (I) $\pi/4$ (J) $2\pi - 1$

$u = x \qquad dv = \sin 2x \, dx$

$du = dx \qquad v = -\frac{1}{2}\cos 2x$

$$\int_0^{\pi} x \sin 2x \, dx = \left(-\frac{1}{2} x \cos 2x + \frac{1}{2}\int \cos 2x \, dx\right)\Big|_0^{\pi}$$

$$= \left(-\frac{1}{2} x \cos 2x + \frac{1}{4}\sin 2x\right)\Big|_0^{\pi}$$

$$= -\frac{\pi}{2} \cdot 1 + 0 - [0 + 0] = -\frac{\pi}{2}$$