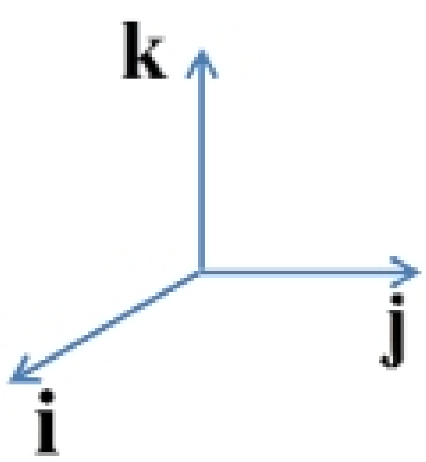
Brown University School of Engineering
ENGN0030: Introduction to Engineering
Homework Set No. 2 Solutions

Due: Thursday September 29 --
By 10:20 in Lecture or
By 4:00pm delivered to Stephanie Gesualdi on 7th floor of Barus & Holley

1. Engineering students have the opportunity to study abroad. Often they do this in their junior year. A popular destination is Australia where there are several fine universities and English is the common language. To get a sense of the distance that you would have to travel if you were to fly from New York City to Sydney Australia, use the solution to Problem 4 of Homework #1 to calculate the flight distance on a 'great circle' route between the two cities. Take the radius of the 'great circle' to be $R = 3,960\ miles$. The longitudes and latitudes of the two cities are:

City	Longitude	Latitude
New York City	$73.782°\ W$	$40.644°\ N$
Sydney, Australia	$151.006°\ E$	$33.901°\ S$



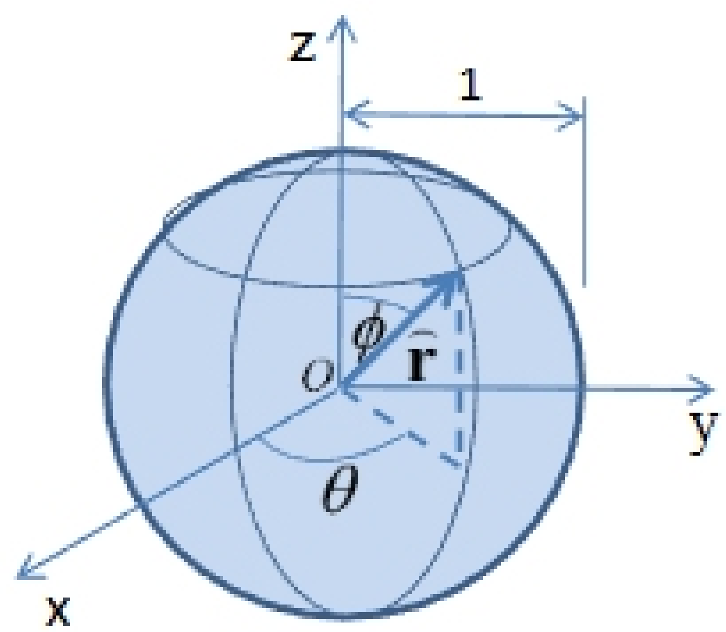


Figure 4: Unit sphere

$$\hat{\mathbf{r}} = (\mathbf{i}\cdot\hat{\mathbf{r}})\mathbf{i} + (\mathbf{j}\cdot\hat{\mathbf{r}})\mathbf{j} + (\mathbf{k}\cdot\hat{\mathbf{r}})\mathbf{k} \ \textit{where}$$
$$(\mathbf{i}\cdot\hat{\mathbf{r}}) = \cos\theta\sin\phi$$
$$(\mathbf{j}\cdot\hat{\mathbf{r}}) = \sin\theta\sin\phi$$
$$(\mathbf{k}\cdot\hat{\mathbf{r}}) = \cos\phi \ \textit{(can be read directly from the figure)}$$
$$\textit{(note that } \sin\phi \textit{ is the length of the projection of } \hat{\mathbf{r}} \textit{ onto the plane } z = 0\textit{)}$$

The great circle distance for a flight from NYC to Sydney Australia, can be obtained by finding the angle β between the position vectors $\mathbf{r}_{NYC}$ and $\mathbf{r}_{Sydney}$ that locate, respectively, New York City and Sydney from the center of the earth. (See Figure below.) In the figure below, z points toward the North Pole. The x-y plane is the equatorial plane. The orientation of the x and y axes is chosen such that the projection of $\mathbf{r}_{NYC}$ onto the equatorial plane lies along the x-axis. The distance along the great circle route is then given by

$$s = R\beta \tag{1.1}$$

where β is in radians. The angle β is obtained from

$$\hat{\mathbf{r}}_{NYC} \cdot \hat{\mathbf{r}}_{Sydney} = \cos\beta. \tag{1.2}$$

The horizontal angle θ and the vertical angles ϕ_{NYC} and ϕ_{Sydney} can be obtained directly from the given table of longitudes and latitudes.

From the solution for Homework 1:

$$\begin{aligned}\hat{\mathbf{r}}_{NYC} &= \cos\theta_{NYC}\sin\phi_{NYC}\mathbf{i} + \sin\theta_{NYC}\sin\phi_{NYC}\mathbf{j} + \cos\phi_{NYC}\mathbf{k} \\ \hat{\mathbf{r}}_{Sydney} &= \cos\theta_{Sydney}\sin\phi_{Sydney}\mathbf{i} + \sin\theta_{Sydney}\sin\phi_{Sydney}\mathbf{j} + \cos\phi_{Sydney}\mathbf{k}.\end{aligned} \tag{1.3}$$

From the given data for longitudes and latitudes, and the choice of axes in the following figure, the angles in (1.3) are:

$$\begin{aligned}&\theta_{NYC} = 0;\quad \theta_{Sydney} = 73.782 + 151.006 = 224.788^\circ \\ &\phi_{NYC} = 90 - 40.644 = 49.356^\circ;\quad \phi_{Sydney} = 90 + 33.901 = 123.901^\circ.\end{aligned} \tag{1.4}$$

Then,

$$\begin{aligned}\hat{\mathbf{r}}_{NYC} &= 0.75877\mathbf{i} + 0.65136\mathbf{k} \\ \hat{\mathbf{r}}_{Sydney} &= (-0.70972)*(0.83001)\mathbf{i} + (-0.70449)*(0.83001)\mathbf{j} + (-0.55776)\mathbf{k}\end{aligned} \tag{1.5}$$

and

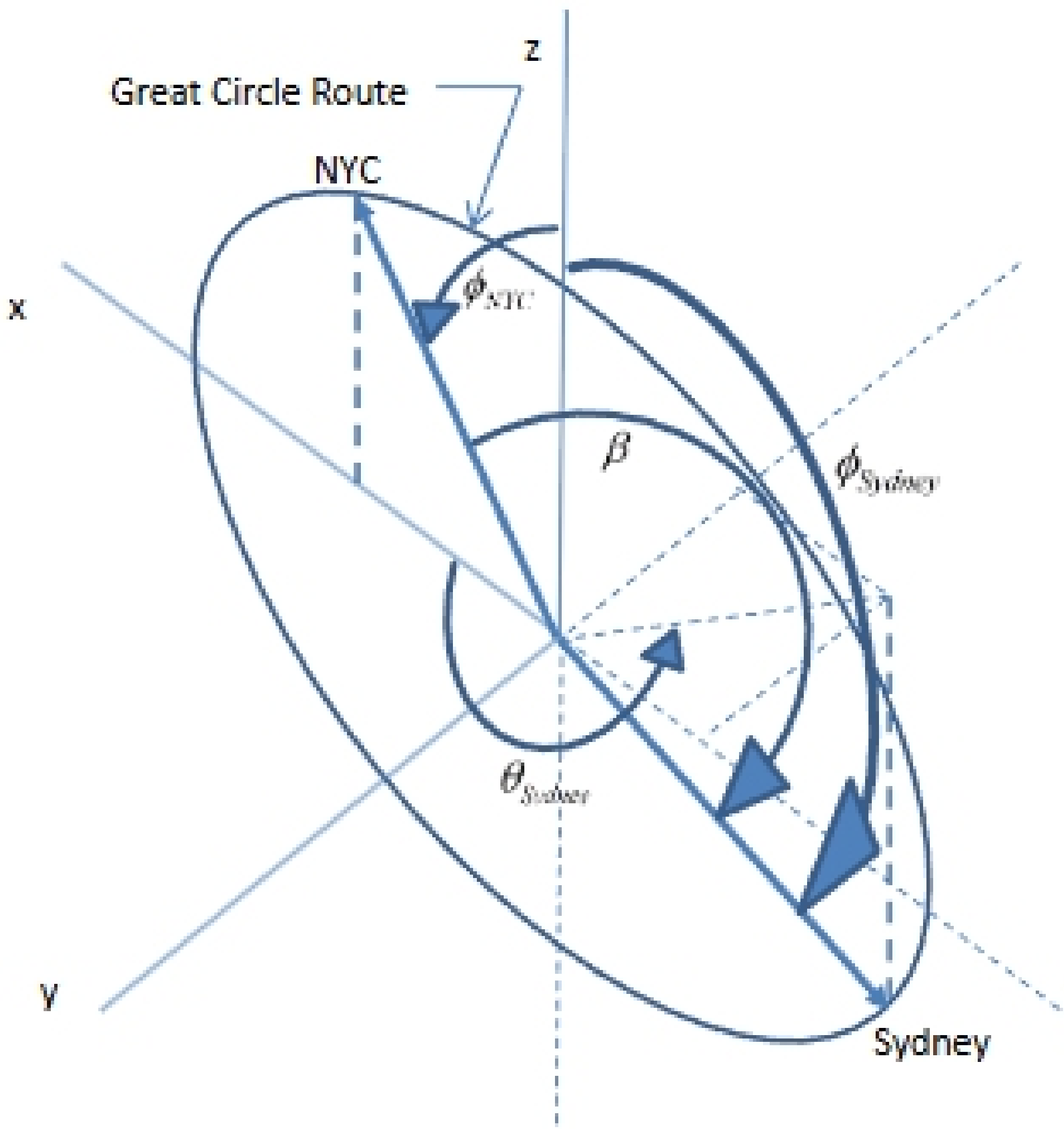
$$\cos\beta = \hat{\mathbf{r}}_{NYC} \cdot \hat{\mathbf{r}}_{Sydney} = 0.75877*(-0.70972)*(0.83001) + 0.65136*(-0.55776) = -0.81027 \tag{1.6}$$

from which,

$$\beta = \cos^{-1}(-0.81028) = 144.124^\circ = 2.5154\ radians. \tag{1.7}$$

Then, from (1.1),

$$s = 3960 * 2.5154 = 9{,}961.1\ miles. \tag{1.8}$$



2. (Prob. 3.17) Each box weighs 40 lb. The angles are measured relative to the horizontal. The surfaces are smooth (i.e. frictionless). Determine the tension in the rope *A* and the normal force exerted on box *B* by the inclined surface. (Tension is the tensile force that tends to stretch the rope.)

Draw a free body diagram for each block. (See below.)

Write equations of equilibrium for each block. In vector form these equations are:

For block A –

$$\mathbf{F}_A + \mathbf{N}_A + \mathbf{F}_C - 40\,\mathbf{j} = 0 \tag{1.9}$$

For block B –

$$-\mathbf{F}_C + \mathbf{N}_B - 40\mathbf{j} = \mathbf{0} \tag{1.10}$$

In component form, relative to the basis **I, j** these equations become :

For block A –