MSE 230 Assignment 7 Solutions Fall 2013

***Problems with an * by them will not be graded, but solutions will be provided.**

1.* 5.6 This problem calls for the mass of hydrogen, per hour, that diffuses through a Pd sheet. It first becomes necessary to employ both Equations (5.1a) and (5.3). Combining these expressions and solving for the mass yields

$$M = JAt = -DAt\frac{\Delta C}{\Delta x}$$

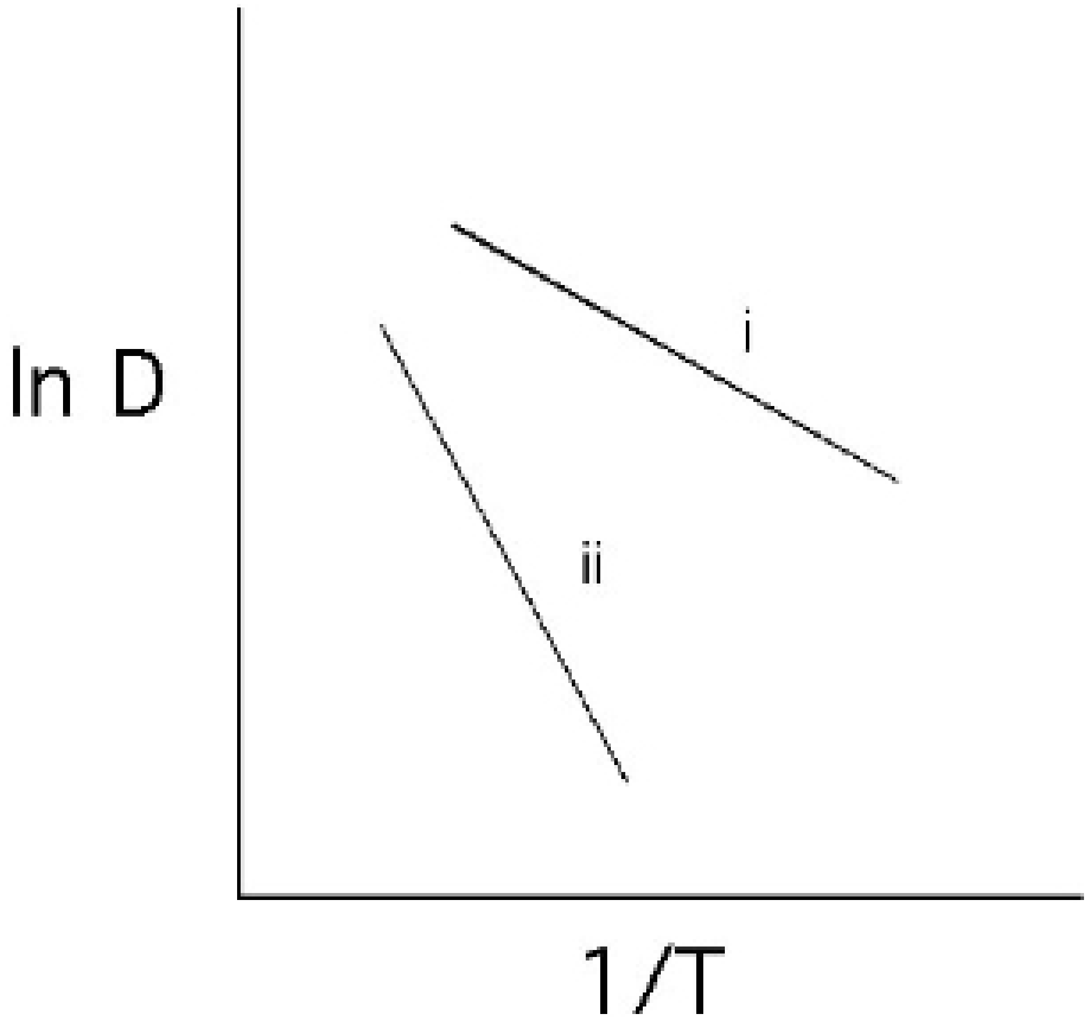
$$= -(1.0 \times 10^{-8}\ \mathrm{m^2/s})(0.2\ \mathrm{m^2})(3600\ \mathrm{s/h})\left[\frac{0.6 - 2.4\ \mathrm{kg/m^3}}{5 \times 10^{-3}\ \mathrm{m}}\right]$$

$$= 2.6 \times 10^{-3}\ \mathrm{kg/h}$$

2.* Carbon should diffuse through iron more rapidly than chromium, and chromium should have a higher activation energy for diffusion.

Carbon is much smaller than iron and may diffuse via interstitial sites. Chromium is close in size to iron and diffuses by a vacancy diffusion mechanism. Therefore, for chromium to diffuse it must exchange lattice positions with a vacant lattice site. The extra requirement for vacancy formation results in a higher activation energy for chromium.

3. **On the same graph**, sketch qualitatively correct plots of ln(D) vs. 1/T for i) carbon diffusing through BCC iron and ii) radioactive iron diffusing through BCC iron. Please provide reasons for any differences between the two plots.



The plot reflects the fact that carbon should diffuse more rapidly through iron than iron diffuses through itself. This is because carbon diffuses through iron via interstitial sites while iron needs vacancies to diffuse.

The higher absolute value of the carbon diffusivity represents the more rapid diffusion rate, while the lower slope for carbon diffusivity represents the lower activation energy required for interstitial diffusion relative to vacancy diffusion.

4. In order to solve this problem, we must first compute the value of $\mathbf{D}_{\mathbf{o}}$ from the data given at 1000 K; this requires the combining of both Equations (5.3) and (5.8). Solving for $\mathbf{D}_{\mathbf{o}}$ from these expressions gives

$$D_o = - \frac{J}{\Delta C/\Delta x}\ \exp$$

$$= - \left(\frac{5.4 \times 10^{-10}\ \mathrm{kg/m^2\text{-}s}}{-\ 350\ \mathrm{kg/m^4}}\right)\ \exp\left[\frac{125000\ \mathrm{J/mol}}{(8.31\ \mathrm{J/mol\text{-}K})(1000\ \mathrm{K})}\right]$$

$$= 5.26 \times 10^{-6}\ \mathrm{m^2/s}$$

The value of the diffusion flux at 1300 K may be computed using these same two equations as follows:

$$J = - D_o\left(\frac{\Delta C}{\Delta x}\right)\ \exp\left(-\frac{Q_d}{RT}\right)$$

$$= - (5.26 \times 10^{-6}\ \mathrm{m^2/s})(-\ 350\ \mathrm{kg/m^4})\ \exp\left[-\frac{125000\ \mathrm{J/mol}}{(8.31\ \mathrm{J/mol\text{-}K})(1300\ \mathrm{K})}\right]$$

$$= 1.74 \times 10^{-8}\ \mathrm{kg/m^2\text{-}s}$$

5.* 5.15 This problem calls for an estimate of the time necessary to achieve a carbon concentration of 0.45 wt% at a point 5 mm from the surface. From Equation (5.6b),

$$\frac{x^2}{Dt} = \text{constant}$$

But since the temperature is constant, so also is **D** constant, and

$$\frac{x^2}{t} = \text{constant}$$

or

$$\frac{x_1^2}{t_1} = \frac{x_2^2}{t_2}$$

Thus,

$$\frac{(2.5\ \text{mm})^2}{10\ \text{h}} = \frac{(5.0\ \text{mm})^2}{t_2}$$

from which

$$t_2 = 40\ \text{h}$$

6. This is a nonsteady-state diffusion situation; thus, it is necessary to employ Equation (6.5), utilizing the following values for the concentration parameters:

$$C_o = 0.0025\ \text{wt\% N}$$
$$C_s = 0.45\ \text{wt\% N}$$
$$C_x = 0.12\ \text{wt\% N}$$

Therefore

$$\frac{C_x - C_o}{C_s - C_o} = \frac{0.12 - 0.0025}{0.45 - 0.0025}$$

$$0.2626 = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

And thus

$$0.7374 = \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Using linear interpolation and the data presented in Table 6.1

z	erf (z)
0.75	0.7112
y	0.7374
0.80	0.7421