Homework 8 - Integration Practice Problems

1) $\int x \sin^2 x \, dx$

$u = x \quad dv = \sin^2 x \, dx = \left(\frac{1-\cos 2x}{2}\right) dx$

$du = dx \quad v = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)$

$$\int x \sin^2 x \, dx = \frac{x}{2}\left(x - \frac{1}{2}\sin 2x\right) - \int \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) dx$$

$$= \frac{x}{2}\left(x - \frac{1}{2}\sin 2x\right) - \frac{1}{2}\left[\left(\frac{x^2}{2}\right) + \frac{1}{4}\cos 2x\right] + C$$

$$= \boxed{\frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + C}$$

2) $\int x 4^x \, dx$

$u = x \quad dv = 4^x \, dx$

$du = dx \quad v = \frac{4^x}{\ln 4}$

$$\int x 4^x \, dx = \frac{x 4^x}{\ln 4} - \int \frac{1}{\ln 4} 4^x \, dx$$

$$= \frac{x 4^x}{\ln 4} - \frac{1}{(\ln 4)^2} 4^x + C$$

$$= \boxed{\frac{4^x}{\ln 4}\left(x - \frac{1}{\ln 4}\right) + C}$$

3) $\int x \sin^{-1} x \, dx$

$u = \sin^{-1} x \quad dv = x\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = \frac{x^2}{2}$

$\int x \sin^{-1} x\,dx = \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$x = \sin\theta \quad dx = \cos\theta\,d\theta$

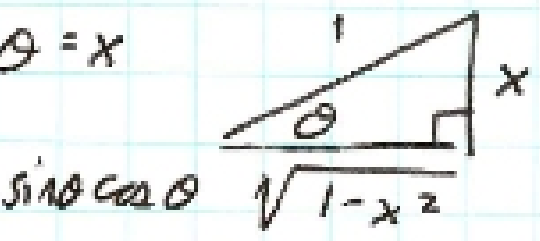
$\int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}} \cos\theta\,d\theta$

$\int \sin^2\theta\,d\theta$

$\int \frac{1-\cos 2\theta}{2}\,d\theta$

$\frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$

$\sin\theta = x$



$\sin 2\theta = 2\sin\theta\cos\theta$

$-\frac{1}{2}$ $\frac{1}{2}\left(\sin^{-1} x - \frac{1}{2}(2)(x)(\sqrt{1-x^2})\right) + C$

$= \boxed{\frac{x^2}{2}\sin^{-1} x - \frac{1}{4}\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + C}$

4) $\int \ln(9x^2-1)\,dx$

$u = \ln(9x^2-1) \quad dv = dx$

$du = \frac{1}{9x^2-1}(18x)\,dx \quad v = x$

$$\int \ln(9x^2-1)\,dx = x\ln(9x^2-1) - \int \frac{18x^2}{9x^2-1}\,dx$$

$$9x^2-1 \overline{)\,18x^2}\;\;2$$

$$\underline{18x^2-2}$$

$$2$$

$$\int \left(2 + \frac{2}{9x^2-1}\right)dx$$

$$2x + 2\int \frac{1}{(3x+1)(3x-1)}\,dx$$

$$\frac{1}{(3x+1)(3x-1)} = \frac{A}{3x+1} + \frac{B}{3x-1}$$

$$1 = A(3x-1) + B(3x+1)$$

Sub $x = \frac{1}{3} \rightarrow 1 = 2B,\ B = \frac{1}{2}$

Sub $x = -\frac{1}{3} \rightarrow 1 = -2A,\ A = -\frac{1}{2}$

$$\int \left[-\frac{1}{2}\left(\frac{1}{3x+1}\right) + \frac{1}{2}\left(\frac{1}{3x-1}\right)\right]dx$$

$$-\frac{1}{6}\ln|3x+1| + \frac{1}{6}\ln|3x-1| + C$$

$$= x\ln(9x^2-1) - \left[2x + 2\left(-\frac{1}{6}\ln|3x+1| + \frac{1}{6}\ln|3x-1|\right)\right] + C$$

$$= x\ln(9x^2-1) - 2x - 2\ln\left[\left|\frac{3x-1}{3x+1}\right|^{\frac{1}{6}}\right] + C$$

$$= \boxed{x\ln(9x^2-1) - 2x - \ln\left(\left|\frac{3x-1}{3x+1}\right|^{\frac{1}{3}}\right) + C}$$