Homework 18 - Sequences

1) a)

n	1	2	3	4
a_n	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$

$a_n = \frac{n}{n+1}$ iv

b)

n	1	2	3	4
a_n	-1	1	-1	1

negative for odd n

$a_n = \cos \pi n$ i

c)

n	1	2	3	4
a_n	1	-1	1	-1

positive for odd n

$a_n = (-1)^{n+1}$ iii

d)

n	1	2	3	4
a_n	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{6}{8}$	$\frac{24}{16}$
	$\frac{1!}{2^1}$	$\frac{2!}{2^2}$	$\frac{3!}{2^3}$	$\frac{4!}{2^4}$

$a_n = \frac{n!}{2^n}$ ii

2) a) $c_n = \frac{3^n}{n!}$ $\boxed{\frac{3}{1}, \frac{9}{2}, \frac{27}{6}, \frac{81}{24} \dots}$

b) $b_n = 5 + \cos \pi n$ $\boxed{4, 6, 4, 6 \dots}$

c) $a_n = \frac{(2n-1)!}{n!}$ $1, \frac{3!}{2!}, \frac{5!}{3!}, \frac{7!}{4!}$

$\boxed{1, 3, 20, 210, \dots}$

3) a) $\frac{1}{1}, -\frac{1}{8}, \frac{1}{27}$ $\quad \frac{(-1)^{n+1}}{n^3}$

b) $\frac{2}{6}, \frac{3}{7}, \frac{4}{8}$ $\quad \frac{n+1}{n+5}$

4 a) $\lim_{n\to\infty} \frac{5n-1}{12n+9} = \frac{5}{12}$ converges

b) $\lim_{n\to\infty} -2^{-n} = 0$ converges

c) $\lim_{n\to\infty} \sqrt{4+\frac{1}{n}} = 2$ converges

d) $\lim_{n\to\infty} \cos^{-1}\left(\frac{n^3}{n^3+1}\right) = \cos^{-1}(1) = 0$ converges

e) $\lim_{n\to\infty} 10 + \left(-\frac{1}{9}\right)^n = 10$ converges

f) $\lim_{n\to\infty} 1{,}01^n = \infty$ diverges

g) $\lim_{n\to\infty} 2^{\frac{1}{n}} = 1$ converges

h) $\lim_{n\to\infty} \frac{n!}{9^n} = \infty$ diverges

i) $\lim_{n\to\infty} \frac{3n^2+n+2}{2n^2-3} = \frac{3}{2}$ converges

j) $\lim_{n\to\infty} \frac{\cos n}{n} = 0$ converges

k) $\lim_{n\to\infty}(\ln 5^n - \ln n!) = \lim_{n\to\infty} \ln \frac{5^n}{n!} = -\infty$ divergent

type $\infty - \infty$
indeterminate

$\lim_{n\to\infty} \frac{5^n}{n!} = 0$

l) $\lim_{n\to\infty}\left(2+\frac{4}{n^2}\right)^{\frac{1}{3}} = 2^{\frac{1}{3}}$ convergent

m) $\lim_{n\to\infty} \ln\left(\frac{2n+1}{3n+4}\right) = \ln\left(\frac{2}{3}\right)$ convergent

n) $\lim_{n\to\infty} \frac{e^n}{2^n} = \infty$ divergent

o) $\lim_{n\to\infty} \frac{e^n+(-8)^n}{5^n} = \lim_{n\to\infty}\left(\frac{e^n}{5^n}+\frac{(-8)^n}{5^n}\right)$ divergent

limit does not exist

p) $\lim_{n\to\infty} n \sin\frac{\pi}{n} = \lim_{n\to\infty} \frac{\sin\frac{\pi}{n}}{\frac{1}{n}} \overset{H}{=} \lim_{n\to\infty} \frac{\left(\cos\frac{\pi}{n}\right)\left(-\frac{\pi}{n^2}\right)}{-\frac{1}{n^2}}$

type $\infty \cdot 0$
indeterminate

type $\frac{0}{0}$

$= \pi$ convergent

q) $\lim_{n\to\infty} \frac{3-4^n}{2+7\cdot 4^n} = \lim_{n\to\infty} \frac{-4^n}{7\cdot 4^n} = -\frac{1}{7}$ convergent

r) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

type 1^∞
indeterminate

$y = \left(1+\frac{1}{x}\right)^x$

$\ln y = x \ln\left(1+\frac{1}{x}\right)$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ type $\frac{0}{0}$

$\overset{H}{=} \lim_{x\to\infty} \frac{\frac{x}{x+1}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = 1$