

# Homework 15 - Review of Limits to Infinity

$$1) \lim_{x \rightarrow \infty} e^x = \infty$$

$$2) \lim_{x \rightarrow \infty} e^{-x} = 0$$

$$3) \lim_{x \rightarrow \infty} (0.2)^x = 0$$

$$4) \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$5) \lim_{x \rightarrow \infty} \ln x = \infty$$

$$6) \lim_{x \rightarrow \infty} \frac{1}{\ln x} = \lim_{u \rightarrow \infty} \frac{1}{u} = 0$$

$$7) \lim_{x \rightarrow \infty} \ln\left(\frac{1}{x}\right) = \lim_{u \rightarrow 0^+} \ln u = -\infty$$

$$8) \lim_{x \rightarrow \infty} \frac{x}{x^2+2} = \lim_{x \rightarrow \infty} \frac{x}{x^2} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$9) \lim_{x \rightarrow \infty} \frac{x}{\ln x} = \infty$$

$$10) \lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$$

$$11) \lim_{x \rightarrow \infty} \frac{x^2+2}{x+e^x} = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} = 0$$

$$12) \lim_{x \rightarrow \infty} \frac{x}{x+1} = \lim_{x \rightarrow \infty} \frac{x}{x} = 1$$

$$13) \lim_{x \rightarrow \infty} \frac{x}{x+e^x} = \lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$$

$$14) \lim_{x \rightarrow \infty} \frac{e^x + \ln x + x + x^3}{x^4 + \ln x + 10^x} = \lim_{x \rightarrow \infty} \frac{e^x}{10^x} = 0$$

$$15) \lim_{x \rightarrow \infty} \sin x \text{ does not exist}$$

$$16) \lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right) = \lim_{u \rightarrow 0} \sin u = 0$$

$$17) \lim_{x \rightarrow \infty} \sin\left(\frac{x+1}{x}\right) = \lim_{x \rightarrow \infty} \sin\left(\frac{x}{x}\right) = \sin 1$$

$$18) \lim_{x \rightarrow \infty} x \sin x \text{ does not exist}$$

$$19) \lim_{x \rightarrow \infty} \frac{1}{x} \sin x = 0$$

$$20) \lim_{x \rightarrow \infty} e^{\left(\frac{1}{x}\right)} = \lim_{u \rightarrow 0} e^u = 1$$

$$21) \lim_{x \rightarrow \infty} x \ln\left(\frac{1}{x}\right) = -\infty \quad (\text{type } \infty \cdot (-\infty))$$

$$22) \lim_{x \rightarrow \infty} (e^{2x} - e^x) = \lim_{x \rightarrow \infty} (e^x(e^x - 1)) = \infty$$

+ type  $(\infty - \infty)$                       + type  $\infty \cdot \infty$

$$23) \lim_{x \rightarrow \infty} e^x \ln\left(\frac{1}{x}\right) = -\infty$$

+ type  $\infty \cdot (-\infty)$

$$24) \lim_{x \rightarrow \infty} x^{\left(\frac{1}{x}\right)} \rightarrow \text{type } \infty^0$$

$$y = x^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln x$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln x}{x} \quad \text{+ type } \frac{\infty}{\infty}$$

$$\stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{1}{x}$$

$$= 0$$

$$\lim_{x \rightarrow \infty} \ln y = 0 \rightarrow \lim_{x \rightarrow \infty} y = e^0 = 1 \rightarrow \lim_{x \rightarrow \infty} x^{\frac{1}{x}} = 1$$

$$25) \lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^x \quad \text{type } 1^\infty$$

$$y = \left(\frac{x}{x+1}\right)^x$$

$$\ln y = x \ln \left(\frac{x}{x+1}\right)$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \left[ \frac{\ln \left(\frac{x}{x+1}\right)}{\frac{1}{x}} \right] \quad \text{type } \frac{0}{0}$$

$$\stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{x+1}{x} \left( \frac{x+1-x}{(x+1)^2} \right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} -\frac{x}{x+1}$$

$$= -1$$

$$\lim_{x \rightarrow \infty} \ln y = -1$$

$$\lim_{x \rightarrow \infty} y = e^{-1}$$

$$\lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^x = \boxed{e^{-1}}$$

$$26) \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^x = \boxed{0} \quad (\text{type } 0^\infty)$$