Homework 5 - Trigonometric Integrals

1) $\int \cos^3 x\, dx$

$= \int \cos^2 x \cos x\, dx$

$= \int (1-\sin^2 x)\cos x\, dx$

$u = \sin x \quad du = \cos x\, dx$

$= \int (1-u^2)\, du$

$= u - \frac{u^3}{3} + C$

$= \boxed{\sin x - \frac{\sin^3 x}{3} + C}$

2) $\int \sin^3 t \cos^3 t\, dt$

$= \int \sin^3 t \cos^2 t \cos t\, dt$

$= \int \sin^3 t (1-\sin^2 t)\cos t\, dt$

$u = \sin t \quad du = \cos t\, dt$

$= \int u^3(1-u^2)\, du$

$= \int (u^3 - u^5)\, du$

$= \frac{u^4}{4} - \frac{u^6}{6} + C$

$= \boxed{\frac{\sin^4 t}{4} - \frac{\sin^6 t}{6} + C}$

3) $\int \tan^3 x \sec x\, dx$

$= \int \tan^2 x \tan x \sec x\, dx$

$= \int (\sec^2 x - 1)\tan x \sec x\, dx$

$u = \sec x \quad du = \tan x \sec x\, dx$

$= \int (u^2 - 1)\, du$

$= \frac{u^3}{3} - u + C$

$= \boxed{\frac{\sec^3 x}{3} - \sec x + C}$

4) $\int \tan x \sec^2 x\, dx$

$u = \tan x \quad du = \sec^2 x\, dx$

$= \int u\, du$

$= \frac{u^2}{2} + C$

$= \boxed{\frac{\tan^2 x}{2} + C}$

5) $\int \sin^4(3x)\,dx$

$= \int [\sin^2(3x)]^2\,dx$

$= \int \left[\frac{1-\cos 6x}{2}\right]^2 dx$

$= \frac{1}{4}\int (1 - 2\cos 6x + \cos^2 6x)\,dx$

$= \frac{1}{4}\int \left(1 - 2\cos 6x + \frac{(1+\cos 12x)}{2}\right) dx$

$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 6x + \frac{1}{2}\cos 12x\right) dx$

$= \frac{1}{4}\left(\frac{3}{2}x - \frac{2}{6}\sin 6x + \frac{1}{24}\sin 12x\right) + C$

$= \boxed{\frac{3}{8}x - \frac{1}{12}\sin 6x + \frac{1}{96}\sin 12x + C}$

6) $\int \sin^4 x \cos^2 x\,dx$

$= \int (\sin^2 x)^2 \cos^2 x\,dx$

$= \int \left[\frac{1-\cos 2x}{2}\right]^2 \left[\frac{1+\cos 2x}{2}\right] dx$

$= \frac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1+\cos 2x)\,dx$

$= \frac{1}{8}\int (1 + \cos 2x - 2\cos 2x - 2\cos^2 2x + \cos^2 2x + \cos^3 2x)\,dx$

$= \frac{1}{8}\int (1 - \cos 2x - \cos^2 2x + \cos^2 2x \cos 2x)\,dx$

$= \frac{1}{8}\left[\int \left[1 - \cos 2x - \left(\frac{1+\cos 4x}{2}\right)\right] dx + \int (1-\sin^2 2x)\cos 2x\,dx\right]$

$u = \sin 2x \quad du = 2\cos 2x\,dx$

$= \frac{1}{8}\left[\int \left(\frac{1}{2} - \cos 2x - \frac{1}{2}\cos 4x\right) dx + \int (1-u^2)\left(\frac{1}{2}\right) du\right]$

$= \frac{1}{8}\left[\frac{1}{2}x - \frac{1}{2}\sin 2x - \frac{1}{8}\sin 4x\right] + \frac{1}{16}\left[u - \frac{u^3}{3}\right] + C$

$= \frac{1}{16}x - \frac{1}{16}\sin 2x - \frac{1}{64}\sin 4x + \frac{1}{16}\sin 2x - \frac{1}{48}\sin^3 2x + C$

$= \boxed{\frac{1}{16}x - \frac{1}{64}\sin 4x - \frac{1}{48}\sin^3 2x + C}$

7) $\int_0^{2\pi} \sin^2 x \, dx$

$= \int_0^{2\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\Big|_0^{2\pi}$

$= \frac{1}{2}(2\pi)$

$= \boxed{\pi}$

8) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^4 x \, dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \sec^2 x \, dx$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x \quad du = \sec^2 x \, dx$

$= \int_{-1}^{1} (u^2 + 1) \, du$

$= \frac{u^3}{3} + u \Big|_{-1}^{1}$

$= \frac{1}{3} + 1 - \left(-\frac{1}{3} - 1\right)$

$= \boxed{\frac{8}{3}}$