Homework 14 - Taylor Polynomials

1) $f(x) = \frac{1}{1+x}$ $a=2$ $f(a) = \frac{1}{3}$ $c_0 = \frac{1}{3}$

$f'(x) = -\frac{1}{(1+x)^2}$ $f'(a) = -\frac{1}{9}$ $c_1 = -\frac{1}{9}$

$f''(x) = \frac{2}{(1+x)^3}$ $f''(a) = \frac{2}{27}$ $c_2 = \frac{2}{27(2)} = \frac{1}{27}$

$f'''(x) = \frac{-6}{(1+x)^4}$ $f'''(a) = \frac{-6}{81} = -\frac{2}{27}$ $c_3 = -\frac{2}{27 \cdot 3!} = -\frac{1}{81}$

$T_2(x) = \frac{1}{3} - \frac{1}{9}(x-2) + \frac{1}{27}(x-2)^2$

$T_4(x) = \frac{1}{3} - \frac{1}{9}(x-2) + \frac{1}{27}(x-2)^2 - \frac{1}{81}(x-2)^3$

2) $f(x) = \frac{1}{1+x}$ $a = -2$

referring to derivatives in 1)

$f(a) = -1$ $c_0 = -1$

$f'(a) = -1$ $c_1 = -1$

$f''(a) = -2$ $c_2 = -1$

$f'''(a) = -6$ $c_3 = -1$

$T_2(x) = -1 - (x+2) - (x+2)^2$

$T_3(x) = -1 - (x+2) - (x+2)^2 - (x+2)^3$

3) $f(x) = \tan x \quad a = 0 \quad f(a) = 0 \quad c_0 = 0$

$f'(x) = \sec^2 x \quad f'(a) = 1 \quad c_1 = 1$

$f''(x) = 2\sec x(\tan x \sec x)$

$= 2\tan x \sec^2 x \quad f''(a) = 0 \quad c_2 = 0$

$f'''(x) = 2\sec^2 x \sec^2 x + 2\tan x(2\sec x)(\tan x \sec x)$

$= 2\sec^4 x + 4\tan^2 x \sec^2 x$

$f'''(a) = 2 \quad c_3 = \frac{2}{3!} = \frac{1}{3}$

$T_2(x) = 0 + 1(x-0) + 0(x-0)^2$

$\boxed{T_2(x) = x}$

$T_3(x) = 0 + 1(x-0) + 0(x-0)^2 + \frac{1}{3}(x-0)^3$

$\boxed{T_3(x) = x + \frac{1}{3}x^3}$

4) $f(x) = \cos x \quad a = 0$

i	$f^i(x)$	$f^i(0)$	$c_i = \frac{f^i(0)}{i!}$
0	$\cos x$	1	1
1	$-\sin x$	0	0
2	$-\cos x$	-1	$-\frac{1}{2}$
3	$\sin x$	0	0
4	$\cos x$	1	$\frac{1}{4!}$

$$c_{2n} = (-1)^n \left(\frac{1}{(2n)!}\right)$$

$$T_{2n}(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \cdots + (-1)^n \frac{x^{2n}}{(2n)!}$$

5) $f(x) = \cos(x^2)$

using 4)...

Maclaurin polynomial: $1 - \frac{1}{2}(x^2)^2 + \frac{1}{4!}(x^2)^4 - \cdots + (-1)^n \frac{(x^2)^{2n}}{(2n)!}$

$$T_{4n} = 1 - \frac{1}{2}x^4 + \frac{1}{4!}x^8 - \cdots + (-1)^n \frac{x^{4n}}{(2n)!}$$