

Homework 9 - Numerical Integration

$$1) \int_0^1 e^{-x^2} dx \quad N=5 \quad \Delta x = \frac{1}{5}$$

endpoints for T_5 : 0, 0.2, 0.4, 0.6, 0.8, 1

$$T_5 = \frac{1}{5} \left(\frac{1}{2} \right) \left[e^{-0^2} + 2(e^{-0.2^2}) + 2(e^{-0.4^2}) + 2(e^{-0.6^2}) + 2(e^{-0.8^2}) + e^{-1^2} \right]$$

$$T_5 \approx 0.78116$$

midpoints for M_5 : 0.1, 0.3, 0.5, 0.7, 0.9

$$M_5 = \frac{1}{5} (e^{-0.1^2} + e^{-0.3^2} + e^{-0.5^2} + e^{-0.7^2} + e^{-0.9^2})$$

$$M_5 \approx 0.72773$$

$$2) \int_0^3 \frac{dx}{x^4+1} \quad N=6 \quad \Delta x = \frac{3}{6} = \frac{1}{2}$$

endpoints for S_6 : 0, 0.5, 1, 1.5, 2, 2.5, 3

$$S_6 = \frac{1}{2} \left(\frac{1}{3} \right) \left[\frac{1}{0^4+1} + \frac{4}{0.5^4+1} + \frac{2}{1^4+1} + \frac{4}{1.5^4+1} + \frac{2}{2^4+1} + \frac{4}{2.5^4+1} + \frac{1}{3^4+1} \right]$$

$$S_6 \approx 1.10903$$

$$3) f_{\text{ave}} = \frac{\int_a^b f(x) dx}{b-a}$$

$$\int_0^3 f(x) dx \approx S_{12}$$
$$\Delta x = \frac{1}{4}$$

$$S_{12} = \frac{\frac{1}{4}}{3} \left[21 + 4(21.3) + 2(21.5) + 4(21.8) + 2(21.6) \right. \\ \left. + 4(21.2) + 2(20.8) + 4(20.6) \right. \\ \left. + 2(20.9) + 4(21.2) + 2(21.1) \right. \\ \left. + 4(21.3) + 21.2 \right]$$

$$S_{12} = 63.6\bar{3}$$

$$f_{\text{ave}} \approx \frac{63.6\bar{3}}{3} = \boxed{21.21^\circ\text{C}}$$

4) Let $T =$ time to travel 1000 miles

Speed = $\sqrt{15f(x)}$ where $f(x)$ is depth in feet

for a small distance Δx , the time $\Delta T \approx \frac{1}{\text{Speed}} \Delta x$

$$\Delta T \approx \frac{1}{\sqrt{15f(x)}} \Delta x$$

$$T = \int_0^{1000} \frac{1}{\sqrt{15f(x)}} dx$$

$$T \approx S_{10} \quad N=10 \quad \Delta x=100$$

$$S_{10} = \frac{100}{3} \left[\frac{1}{\sqrt{15(13000)}} + \frac{4}{\sqrt{15(11500)}} + \frac{2}{\sqrt{15(10500)}} \right.$$

$$+ \frac{4}{\sqrt{15(9000)}} + \frac{2}{\sqrt{15(8500)}}$$

$$+ \frac{4}{\sqrt{15(7000)}} + \frac{2}{\sqrt{15(6000)}}$$

$$+ \frac{4}{\sqrt{15(4400)}} + \frac{2}{\sqrt{15(3800)}}$$

$$+ \left. \frac{4}{\sqrt{15(3200)}} + \frac{1}{\sqrt{15(2000)}} \right]$$

$$T \approx S_{10} \approx \boxed{3.347 \text{ hours}}$$