

FIN 423 – Fall 2012

Answers to Homework Set 2 (Chapters 7 to 8)

Chapter 7- Bond Price Volatility

Multiple-choice theory questions (Each theory question is worth 0.5 point)

1. Which one of the following measures a bond's sensitivity to changes in market interest rates?
 - A. yield to call
 - B. yield to market
 - C. duration**
 - D. immunization
 - E. target date valuation

2. A change in a bond's price caused by which one of the following is defined as the dollar value of an 01?
 - A. change in yield to call due to passage of one year
 - B. change in yield to maturity of one percent
 - C. change in yield to maturity of one basis point**
 - D. change in coupon rate of one percent
 - E. change in coupon rate of one basis point

- 3.. Which combination of bond characteristics causes a bond to be most sensitive to changes in market interest rates?
 - I. low coupon rates
 - II. high coupon rates
 - III. short time to maturity
 - IV. long time to maturity
 - A. III only
 - B. I and III only
 - C. I and IV only**
 - D. II and III only
 - E. II and IV only

4. Which one of the following statements is correct concerning Macaulay duration?
 - A. The Macaulay duration of a zero coupon bond is equal to the time to maturity. (Let $C=0$ in Equation (5) on page 148 and notice that what will be left on the numerator is $nM/(1+y)^n$, which equals to nP . We can simplify by P and get Macaulay duration = n)**
 - B. Most bonds have Macaulay durations in excess of 15 years.
 - C. The Macaulay duration of a coupon bond is a linear function between the time to maturity and the duration.
 - D. The Macaulay duration of a coupon bond is greater than that of a zero coupon bond given equal maturity dates.

E. The percentage change in a bond's price is approximately equal to the change in the yield to maturity multiplied by $(-1 \times \text{Macaulay duration})$.

Quantitative questions

Assume that each coupon bond pays interest semiannually.

Price value of a basis point or Dollar value of an 01

5. Consider a 6% bond with a par value of \$100 and a yield to maturity of 7% that matures in 12 years. What is the dollar value of an 01 for this bond? (0.5 point)

Initial price:

$$\text{PMT} = (0.06 \times \$100) \div 2 = 3$$

$$\text{I/Y} = 7\% \div 2 = 3.5$$

$$\text{N} = 12 \times 2 = 24$$

$$\text{FV} = 100$$

$$\text{CPT PV} = -91.9708; \text{Initial Price} = \$91.9708$$

Price if yield change by 1 basis point (for example, the yield increases by 1 basis point; it does not matter whether we do the calculations with an increase or a decrease in yield by such a small change):

$$\text{PMT} = 3$$

$$\text{I/Y} = 7.01\% \div 2 = 3.505$$

$$\text{N} = 24$$

$$\text{FV} = 100$$

$$\text{CPT PV} = -91.8948; \text{New Price} = \$91.8948$$

$$\text{Price value of a basis change} = |\$91.8948 - \$91.9708| = \$0.076$$

Yield value of a price change

6. A 4-year 5.8% coupon bond has a yield to maturity of 7% and a par value of \$100.

a. What is the price of this bond? (0.25 point)

$$\text{N} = 4 \times 2 = 8$$

$$\text{PMT} = (0.058 \times 100) \div 2 = 2.9$$

$$\text{I/Y} = 7\% \div 2 = 3.5$$

$$\text{FV} = 100$$

$$\text{CPT PV} = -95.8756; \text{Price of the bond} = \$95.8756$$

b. If the price decreases by \$5, what is the new yield to maturity? (0.5 point)

$$\text{New Price} = \$90.8756$$

$$\text{N} = 8$$

$$\text{PMT} = 2.9$$

$$\text{FV} = 100$$

$$\text{PV} = -90.8756$$

$$\text{CPT I/Y} = 4.2703; \text{New Yield} = 4.2703 \times 2 = 8.5408\%$$

c. Estimate the yield value of a 5-dollar price change. (0.5 point)

$$\text{Yield value of a 5-dollar price change} = |8.5408\% - 7\%| = 1.5408\%$$

Macaulay duration and Modified duration

7. A 8% bond with a \$100 par value sells at par and has a 8% yield to maturity. The bond matures in 2 years.

a. Using Equation (5) on page 148 of the Textbook, calculate the Macaulay duration of this bond in half years and in years. (0.5 point + 0.25 point)

$$C = (0.08 \times \$100) \div 2 = \$4$$

$$y = 0.08 \div 2 = 0.04$$

$$P = \$100 = M$$

$$n = 2 \times 2 = 4$$

$$\text{Macaulay duration}(\text{in half years}) = \frac{\sum_{t=1}^4 \frac{tC}{(1+y)^t} + \frac{4M}{(1+y)^4}}{P}$$

$$\text{Macaulay duration}(\text{in half years}) = \frac{\frac{1C}{(1+y)} + \frac{2C}{(1+y)^2} + \frac{3C}{(1+y)^3} + \frac{4C}{(1+y)^4} + \frac{4M}{(1+y)^4}}{P}$$

$$\text{in} \frac{\frac{\$4}{(1+0.04)} + \frac{2 \times \$4}{(1+0.04)^2} + \frac{3 \times \$4}{(1+0.04)^3} + \frac{4 \times \$104}{(1+0.04)^4}}{\$100} = \frac{3.8462 + 7.3964 + 10.6680 + 355.5985}{100}$$

$$\text{in} \frac{377.5091}{100}$$

$$\text{Macaulay duration}(\text{in half years}) = 3.775091 = 3.78$$

$$\text{Macaulay duration}(\text{in years}) = \frac{3.775091}{2} = 1.8875 = 1.89$$

b. Calculate the modified duration of this bond in years. (0.5 point)

$$\text{Modified duration}(\text{in years}) = \frac{1.89}{1+0.04} = 1.81$$

8. A \$1,000 par value corporate bond has a yield to maturity of 7%. It matures in 12 years. Its coupon rate is 6%.

a. What is the current price of this bond? What is the quoted price as a percentage of the par value? (0.25 point + 0.25 point)

$$FV = 1000$$

$$I/Y = 7\% \div 2 = 3.5$$

$$N = 12 \times 2 = 24$$

$$PMT = (0.06 \times \$1,000) \div 2 = 30$$

$$\text{CPT PV} = -919.7082; \text{ Price of the bond} = \mathbf{\$919.7082}$$

Quoted price = 91.9708 (a percentage of par)