

Often the biggest challenge to doing homework is to figure out the detailed steps. A more important task is to think about what you learned. After doing a problem, identify the big ideas and the details. If you cannot identify what you learned immediately after doing a problem, you will certainly not be able to recall the ideas on a test.

1. **Mizar, the first binary star discovered from the spectrum.** Even though Mizar appears to be a single star, Pickering's spectrum showed it to be a binary star. You will need to refer to the spectrum shown in class on Oct. 25th. The speed of light is 3×10^5 km/s.

- a. (2 pts.) Look at the drawing of the orbit of the binary star. Be certain to note the location of the earth. Why is there only a single spectral line on 1 and 23 October and two spectral lines on other days?

On Oct 1 & 23, the motion is perpendicular to the line of sight, and the Doppler Effect is sensitive only to motion parallel to the line of sight.

- b. (2 pts.) Why is the H β line of hydrogen not at its laboratory wavelength even on 1 October?

Even though the orbital motion is perpendicular to the line of sight on that day, there is a component of the motion of the entire binary star system that is parallel to the line of sight.

- c. (1 pt.) Find the speed of the center of the binary star system.

Pick the motion on 1 Oct, when the Doppler Effect is entirely due to the motion of the star system.

$$v = c (\lambda_{\text{obs}}/\lambda_{\text{lab}} - 1) = 3 \times 10^5 \text{ km/s} (4861.13/4861.30 - 1) = -10 \text{ km/s (toward us)}$$

- d. (1 pt.) Find the orbital speed of one of the stars.

The relative speed of the two stars on 12 Oct yields twice the orbital speed.

$$2v_{\text{orbital}} = c (\lambda_{\text{obsA}}/\lambda_{\text{lab}} - 1) - c (\lambda_{\text{obsB}}/\lambda_{\text{lab}} - 1) = c (\lambda_{\text{obsA}} - \lambda_{\text{obsB}})/\lambda_{\text{lab}} = 3 \times 10^5 \text{ km/s} (2/4861.30)$$

$$v_{\text{orbital}} = 62 \text{ km/s}$$

- e. (0 pts.) What are the big ideas needed to answer this question? (You may want to do this part first.)

(1) The Doppler Effect is sensitive only to the component of the motion towards or away from the observer. (2) The Doppler formula relates the wavelengths observed and emitted and the speed of the source.

2. **Discovery of the first quasar.** In 1962, Maarten Schmidt observed the spectrum of an object that emitted radio waves and visually looked like a star. Since stars do not emit radio waves, this was a very unusual object. He observed the part of the spectrum from 4860 Å to 6030 Å. He found spectral lines at 5571 Å and 4974 Å. You are going to figure out the redshift.

- a. Hint: the same factor $\lambda_{\text{obs}}/\lambda_{\text{lab}}$ applies to every spectral line. (2 pts.) Explain why.

There are two possible explanations. (1) According to Doppler, the factor $\lambda_{\text{obs}}/\lambda_{\text{lab}}$ is related to the speed of the source, which of course is the same for every spectral line in the source. (2) If the wavelength changes because of the expansion of the universe, the factor $\lambda_{\text{obs}}/\lambda_{\text{lab}}$ is the factor by which the universe expanded, which of course is the same for every spectral line in the source.

- b. (3 pts.) Assume OII (ionized oxygen with one electron removed) accounts for the line at 4974 Å. What then is the redshift?

$$z = 4974/3727 - 1 = 0.3346$$

- c. (1 pt.) Assume the redshift in part (a) is correct. What is the wavelength at which the line H ϵ appears in the spectrum of the object?

$$3970 * 1.3346 = 5298$$

- d. (2 pts.) Why is the identification of the line at 4974 Å as OII incorrect?

M Schmidt did not see a line at 5298. Hydrogen should be present in the spectrum.

- e. (4 pts.) Now you know the line at 4974 Å is not due to OII. Identify (determine the element and the particular spectral line of that element) these two lines and determine the redshift of this object. Note: Not all of the lines in the table are present in all astronomical objects. However, a line in the hydrogen series cannot occur by itself.

Assume 4974 is due to some other line element and line. Do the same steps.

Alternatively, find the redshift for every assumption for the line at 4974 and the line at 5571: In columns 3 and 8 of the table are the inferred redshift if the line is assumed to be the one observed at 4974. Columns 4 and 9 are the inferred redshift if the line is assumed to be the one observed at 5571. The two observed lines are at the same redshift, 0.146, if 4974 is the H γ line of hydrogen and 5571 is the H β line of hydrogen.

Table 2. Spectral lines of hydrogen and oxygen. Column 2 & 7: laboratory wavelengths. Col3 & 8: redshift if the line is observed at 4974. Col4 & 9: redshift if the line is observed at 5571. Col5 &10: Observed wavelength if z=0.146.

<i>Line</i>	<i>Wavelength</i>	<i>z</i>	<i>z</i>	<i>λ</i>	<i>Line</i>	<i>Wavelength</i>	<i>z</i>	<i>z</i>	<i>λ</i>
		4974	5571	<i>z 0.146</i>			4974	5571	<i>z 0.146</i>
H α	6562	-0.242	-0.151	7521	OII	3727	0.335	0.495	4271
H β	4861	0.023	<u>0.146</u>	5571	OIII	5007	-0.007	0.113	5739
H γ	4340	<u>0.146</u>	0.284	4974					
H δ	4101	0.213	0.358	4700					
H ϵ	3970	0.253	0.403	4550					

- f. (0 pts.) What are the big ideas needed to answer this question? (You may want to do this part first.)

The first big idea is in part (a). The second big idea is how the observed and emitted wavelengths are related to redshift.

Table 1. Spectral lines of hydrogen and oxygen and their laboratory wavelengths. OII means oxygen with one electron removed, and OIII is oxygen with two electrons removed.

<i>Line</i>	<i>Wavelength</i>	<i>Line</i>	<i>Wavelength</i>
H α	6562 Å	OII	3727 Å
H β	4861 Å	OIII	5007 Å
H γ	4340 Å		
H δ	4101 Å		
H ϵ	3970 Å		

3. The present distance to Hoag's Object is 300 Mpc, and its speed is 18,000 km/s. For Hubble's constant, use the value 60 km/s/Mpc (equal to 0.061/Byr). A Mpc is 3.1×10^{19} km. A billion years is 3×10^{16} s.

- a. (3 pts.) A billion years ago, Hoag's Object was moving away from us at about the same speed. What is the reason for that?

Without forces acting on it, Hoag's Object moves at the same speed according to Newton's First Law. This is a useful approximation for distant objects over times short compared to the age of the universe.

- b. (3 pts.) How far from us was Hoag's Object at that time?

$$\begin{aligned} \text{Distance moved} &= \text{speed} \times \text{time} = 18,000 \text{ km/s} \times 3 \times 3 \times 10^{16} \text{ s} = 1.8 \times 9 \times 10^{20} \text{ km} \\ &= 1.8 \times 9 \times 10^{20} \text{ km} / (3.1 \times 10^{19} \text{ km/Mpc}) = 52 \text{ Mpc}. \end{aligned}$$

Alternatively, speed = $H D = 0.061/\text{Byr} \times 300 \text{ Mpc} = 18.3 \text{ Mpc/Byr}$. The distance moved = $18.3 \text{ Mpc/Byr} \times 3 \text{ Byr} = 55 \text{ Mpc}$. (The numbers disagree slightly because of rounding errors: a billion years is 3.16×10^{16} s.)

The distance is therefore $300 - 55 = 245 \text{ Mpc}$

- c. (1 pts.) What was the value of Hubble's constant at that time? Is Hubble's constant a constant (one that does not change with time)?

$$H = v/D = 18000 \text{ km/s} / (245 \text{ Mpc}) = 73 \text{ km/s/Mpc}$$

- d. (0 pts.) What is the big idea needed to answer this question? (You may want to do this part first.)

Part (a) has one big idea. Hubble's Law is the second big idea. The third big idea is that the distance traveled is the product of speed and time.