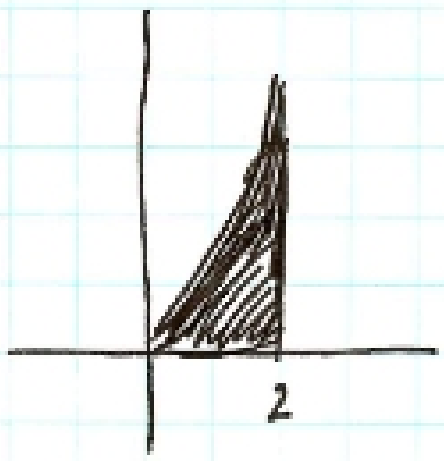
Homework 13

1)



$\bar{x} = \frac{M_y}{m}$ $\quad$ $\bar{y} = \frac{M_x}{m}$

$m = \int_0^2 \rho x^2 \, dx$

$= \rho \frac{x^3}{3} \Big|_0^2$

$= \frac{8\rho}{3}$

$M_y = \int_0^2 \rho x (x^2) \, dx$

$= \rho \frac{x^4}{4} \Big|_0^2$

$\boxed{M_y = 4\rho}$

$\bar{x} = \frac{4\rho}{\left(\frac{8\rho}{3}\right)}$

$\bar{x} = \frac{3}{2}$

$M_x = \int_0^2 \rho \left(\frac{1}{2}\right)(x^2)^2 \, dx$

$= \frac{\rho}{2}\left(\frac{x^5}{5}\right) \Big|_0^2$

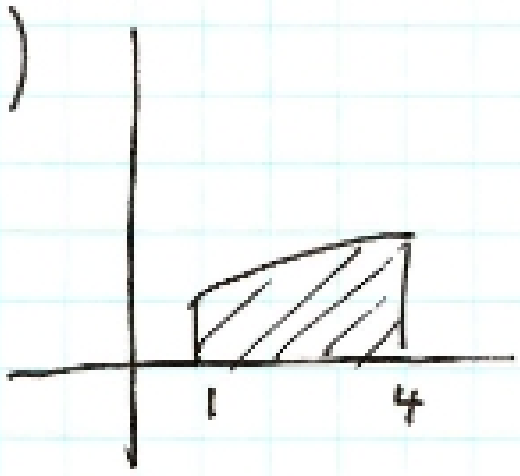
$\boxed{M_x = \frac{16\rho}{5}}$

$\bar{y} = \frac{\left(\frac{16\rho}{5}\right)}{\left(\frac{8\rho}{3}\right)}$

$\bar{y} = \frac{6}{5}$

$\boxed{(\bar{x}, \bar{y}) = \left(\frac{3}{2}, \frac{6}{5}\right)}$

2)



$\bar{x} = \frac{M_y}{m}$ $\quad$ $\bar{y} = \frac{M_x}{m}$

Assume $\rho = 1$

$$m = \int_1^4 \sqrt{x}\,dx$$
$$= \frac{2}{3}x^{3/2}\Big|_1^4$$
$$= \frac{16}{3} - \frac{2}{3}$$
$$= \frac{14}{3}$$

$$M_y = \int_1^4 x\sqrt{x}\,dx$$
$$= \frac{2}{5}x^{5/2}\Big|_1^4$$
$$= \frac{64}{5} - \frac{2}{5}$$
$$= \frac{62}{5}$$

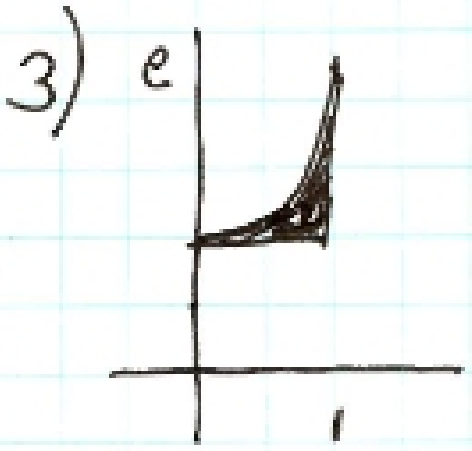
$$\bar{x} = \frac{\left(\frac{62}{5}\right)}{\left(\frac{14}{3}\right)} = \frac{93}{35}$$

$$M_x = \int_1^4 \frac{1}{2}(\sqrt{x})^2\,dx$$
$$= \frac{1}{2}\left(\frac{x^2}{2}\right)\Big|_1^4$$
$$= \frac{1}{2}\left(8 - \frac{1}{2}\right)$$
$$= \frac{15}{4}$$

$$\bar{y} = \frac{\left(\frac{15}{4}\right)}{\left(\frac{14}{3}\right)} = \frac{45}{56}$$

$$(\bar{x}, \bar{y}) = \left(\frac{93}{35}, \frac{45}{56}\right)$$

3)



Assume $\rho = 1$

$\bar{x} = \frac{M_y}{m}$ $\qquad$ $\bar{y} = \frac{M_x}{m}$

$m = \int_0^1 (e^x - 1)\,dx$

$= (e^x - x)\Big|_0^1$

$= e - 1 - 1$

$m = e - 2$

$M_y = \int_0^1 x e^x\,dx$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$= x e^x\Big|_0^1 - \int_0^1 e^x\,dx$

$= (x e^x - e^x)\Big|_0^1$

$= e - e - 0 + 1$

$= 1$

$\bar{x} = \frac{1}{e-2}$

$M_x = \int_0^1 \frac{1}{2}\left((e^x)^2 - 1^2\right)dx$

$= \frac{1}{2}\int_0^1 (e^{2x} - 1)\,dx$

$= \frac{1}{2}\left(\frac{1}{2}e^{2x} - x\right)\Big|_0^1$

$= \frac{1}{2}\left(\frac{1}{2}e^2 - 1 - \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{1}{2}e^2 - \frac{3}{2}\right)$

$= \frac{1}{4}(e^2 - 3)$

$\bar{y} = \frac{\frac{1}{4}(e^2-3)}{e-2}$

$(\bar{x}, \bar{y}) = \boxed{\left(\frac{1}{e-2}, \frac{e^2-3}{4(e-2)}\right)}$