

Homework 31 - Conic Sections

$$1) a) \left(\frac{x}{9}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

ellipse, center $(0, 0)$

$$a = 9 \quad b = 4 \quad c = \sqrt{a^2 - b^2} = \sqrt{81 - 16} = \sqrt{65}$$

vertices: $(-9, 0), (9, 0), (0, 4), (0, -4)$

foci: $(-\sqrt{65}, 0), (\sqrt{65}, 0)$

$$b) \left(\frac{x-3}{7}\right)^2 - \left(\frac{y+1}{4}\right)^2 = 1$$

hyperbola, center $(3, -1)$

$$a = 7, \quad b = 4, \quad c = \sqrt{a^2 + b^2} = \sqrt{49 + 16} = \sqrt{65}$$

vertices: $(10, -1), (-4, -1)$

foci: $(3 - \sqrt{65}, -1), (3 + \sqrt{65}, -1)$

$$2) a) \left(\frac{x}{6}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$b) \left(\frac{x-8}{6}\right)^2 + \left(\frac{y+8}{3}\right)^2 = 1$$

3) center: $(0, 0)$

$$a = 7 \quad b = 5 \quad c = \sqrt{a^2 - b^2} = \sqrt{49 - 25} = \sqrt{24}$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{7}\right)^2 = 1$$

3 b) Center: (0, 0)

$$c = 6 \quad a = 10 \quad c^2 = a^2 - b^2$$

$$b^2 = a^2 - c^2 = 100 - 36$$

$$b^2 = 64$$

$$b = 8$$

$$\left(\frac{x}{10}\right)^2 + \left(\frac{y}{8}\right)^2 = 1$$

4 a) Center: (0, 0)

$$a = 3 \quad c = 5 \quad c^2 = a^2 + b^2$$

$$25 = 9 + b^2$$

$$b = 4$$

$$\left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2 = 1$$

b) Center: (0, -1)

$$a = 5 \quad c = 8 \quad c^2 = a^2 + b^2$$

$$64 = 25 + b^2$$

$$b = \sqrt{39}$$

$$\left(\frac{y+1}{5}\right)^2 - \left(\frac{x}{\sqrt{39}}\right)^2 = 1$$

5 a) center: $(0, 0)$

$$c = 4$$

$$y = \frac{1}{4c} x^2$$

$$y = \frac{1}{16} x^2$$

b) $c = 4$

$$(x-2) = -\frac{1}{16} y^2$$

6 a) $x^2 + 4y^2 = 16$ ellipse

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$$a=4, b=2 \quad c^2 = a^2 - b^2 = 16 - 4$$

$$c = \sqrt{12}$$

Center: $(0, 0)$

vertices: $(-4, 0), (4, 0), (0, -2), (0, 2)$

foci: $(-\sqrt{12}, 0), (\sqrt{12}, 0)$

b) $\left(\frac{x-3}{4}\right)^2 - \left(\frac{y+5}{7}\right)^2 = 1$ hyperbola

center: $(3, -5)$

$$a=4 \quad b=7 \quad c^2 = a^2 + b^2 = 16 + 49 = 65$$

$$c = \sqrt{65}$$

vertices: $(-1, -5), (7, -5)$

foci: $(3 - \sqrt{65}, -5), (3 + \sqrt{65}, -5)$

asymptotes: $y + 5 = \pm \frac{7}{4}(x - 3)$ OR $y = \frac{7}{4}x - \frac{41}{4}$

and $y = -\frac{7}{4}x + \frac{1}{4}$