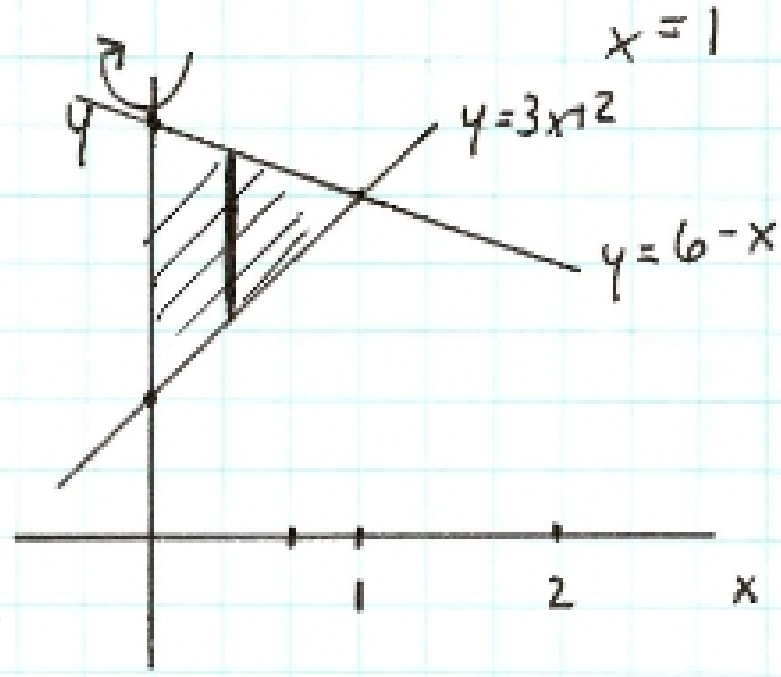
Homework 3 - The Method of Cylindrical Shells

1) intersection: $3x+2 = 6-x$

$4x = 4$

$x = 1$



$V_i = 2\pi r_i h_i \Delta x$

$r_i = x_i \quad h_i = (6-x) - (3x+2)$

$h_i = 4 - 4x$

$V_i = 2\pi x_i (4-4x_i)\Delta x$

$V = \int_0^1 2\pi x(4-4x)\,dx$

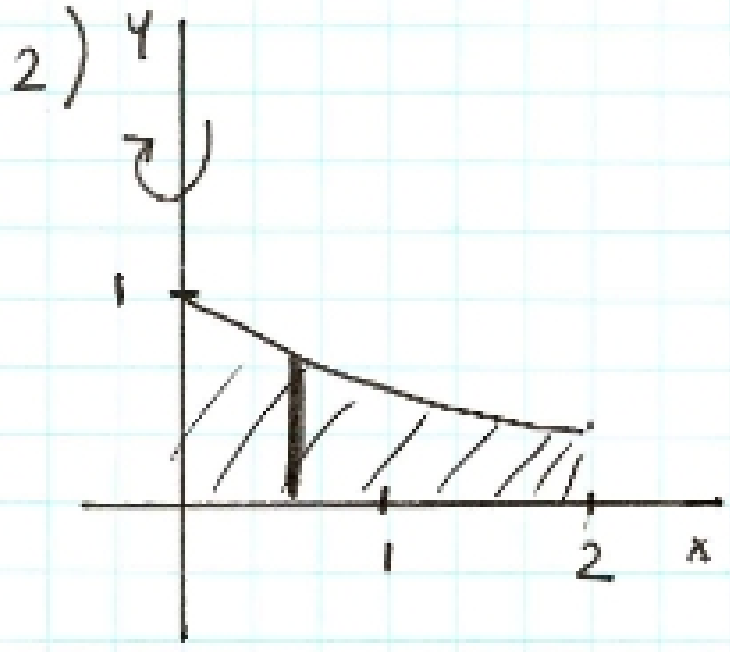
$= \int_0^1 2\pi (4x - 4x^2)\,dx$

$= 2\pi \left(2x^2 - \frac{4x^3}{3}\right)\Big|_0^1$

$= 2\pi \left(2 - \frac{4}{3}\right)$

$= \boxed{\frac{4\pi}{3}}$

2)



$V_i = 2\pi r_i h_i \Delta x$

$r_i = x_i \quad h_i = \frac{1}{\sqrt{x_i^2+1}}$

$V_i = 2\pi x_i \left(\frac{1}{\sqrt{x_i^2+1}}\right)\Delta x$

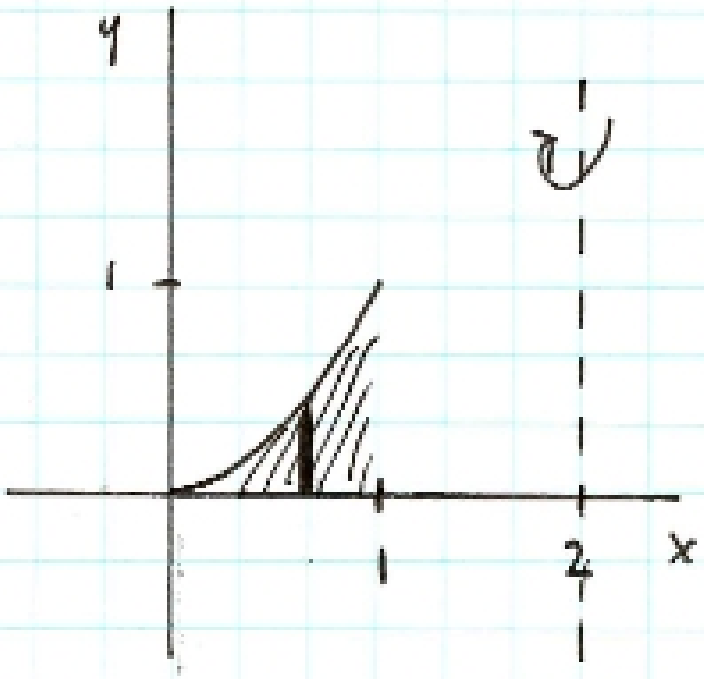
$V = \int_0^2 2\pi x \left(\frac{1}{\sqrt{x^2+1}}\right) dx$

$u = x^2+1 \quad du = 2x\,dx$

$\int_1^5 \pi \frac{du}{\sqrt{u}}$

$2\pi\sqrt{u}\,\Big|_1^5 = \boxed{2\pi(\sqrt{5}-1)}$

3)



Shells:

$V_i = 2\pi r_i h_i \Delta x$

$r_i = 2 - x_i \qquad h_i = x_i^3$

$V_i = 2\pi(2 - x_i) x_i^3 \Delta x$

$V = \int_0^1 2\pi(2-x) x^3 \, dx$

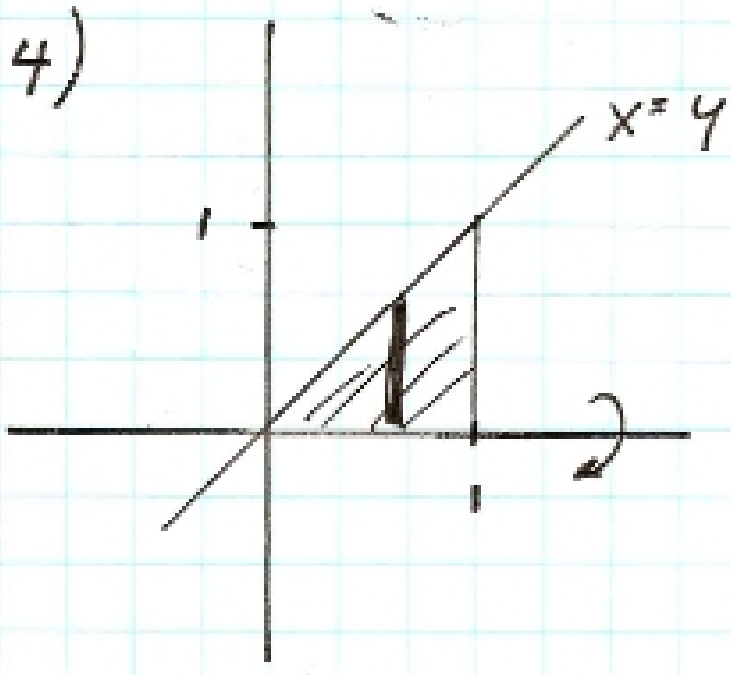
$= 2\pi \int_0^1 (2x^3 - x^4) \, dx$

$= 2\pi \left[\frac{x^4}{2} - \frac{x^5}{5} \right] \Big|_0^1$

$= 2\pi \left[\frac{1}{2} - \frac{1}{5} \right]$

$= 2\pi \left(\frac{3}{10} \right) = \boxed{\frac{3\pi}{5}}$

4)



disks:

$V_i = \pi r_i^2 \Delta x$

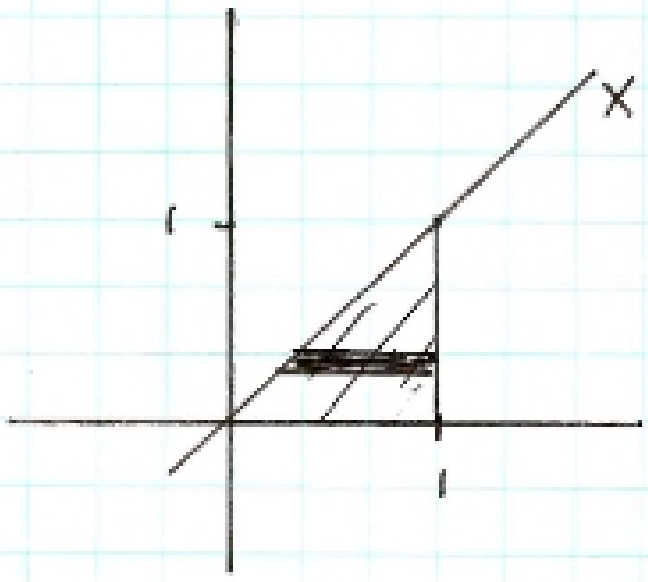
$r_i = x_i$

$V_i = \pi x_i^2 \Delta x$

$V = \int_0^1 \pi x^2 \, dx$

$= \pi \frac{x^3}{3} \Big|_0^1 = \boxed{\frac{\pi}{3}}$

or Shells:



$V_i = 2\pi r_i h_i \Delta y$

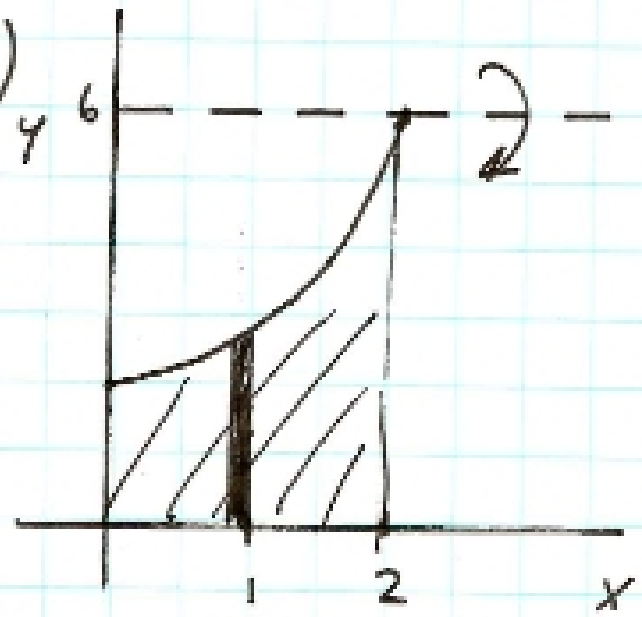
$r_i = y_i \qquad h_i = 1 - y_i$

$V_i = 2\pi y_i (1 - y_i) \Delta y$

$V = \int_0^1 2\pi y (1-y) \, dy$

$= 2\pi \int_0^1 (y - y^2) \, dy = 2\pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1$

$= \boxed{\frac{\pi}{3}}$

4) 

washer method

$r_{outer} = 6 \qquad r_{inner} = 6 - (x^2+2)$

$V_i = \pi\left(6^2 - (6-(x^2+2))^2\right)$

$V = \int_0^2 \pi\left(36 - (36 - 12(x^2+2) + (x^2+2)^2\right) dx$

$= \int_0^2 \pi\left(36 - (36 - 12x^2 - 24 + x^4 + 4x^2 + 4)\right) dx$

$= \int_0^2 \pi\left(-x^4 + 8x^2 + 20\right) dx$

$= \pi\left(-\frac{x^5}{5} + \frac{8x^3}{3} + 20x\right)\Big|_0^2$

$= \pi\left(-\frac{32}{5} + \frac{64}{3} + 40\right)$

$= \pi\left(\frac{-96 + 320 + 600}{15}\right)$

$= \boxed{\pi\left(\frac{824}{15}\right)}$