Homework 28 - Arc Length and Speed

1 a) $(2t^2, 3t^2-1)$, $0 \le t \le 4$

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$S = \int_0^4 \sqrt{(4t)^2 + (6t)^2}\, dt$$

$$= \int_0^4 \sqrt{16t^2 + 36t^2}\, dt$$

$$= \int_0^4 \sqrt{52}\, t\, dt$$

$$= \sqrt{52}\, \frac{t^2}{2} \Big|_0^4$$

$$= 8\sqrt{52}$$

$$= \boxed{16\sqrt{13}}$$

b) $(3t^2, 4t^3)$ $1 \le t \le 4$

$$S = \int_1^4 \sqrt{(6t)^2 + (12t^2)^2}\, dt$$

$$= \int_1^4 \sqrt{36t^2 + 144t^4}\, dt$$

$$= \int_1^4 6t\sqrt{1 + 4t^2}\, dt$$

$u = 1 + 4t^2$ $du = 8t\, dt$

$$= \int_5^{65} \frac{6}{8}\sqrt{u}\, du$$

$$= \frac{6}{8}\left(\frac{2}{3}\right) u^{3/2} \Big|_5^{65}$$

$$= \boxed{\frac{1}{2}\left(65\sqrt{65} - 5\sqrt{5}\right)}$$

1 c) $(\sin 3t, \cos 3t) \quad 0 \le t \le \pi$

$$S = \int_0^{\pi} \sqrt{(3\cos 3t)^2 + (-3\sin 3t)^2}\, dt$$

$$= \int_0^{\pi} 3\sqrt{\cos^2 3t + \sin^2 3t}\, dt$$

$$= \int_0^{\pi} 3\, dt$$

$$= \boxed{3\pi}$$

2) $(\ln(t^2+1), t^3) \quad t=1$

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(\frac{2t}{t^2+1}\right)^2 + (3t^2)^2}$$

$$\text{speed}\Big|_{t=1} = \sqrt{\left(\frac{2}{2}\right)^2 + 3^2}$$

$$= \boxed{\sqrt{10}\ \text{m/s}}$$

3) $c(t) = (t^3 - 4t,\ t^2 + 1)$

$$\text{speed} = \sqrt{(3t^2-4)^2 + (2t)^2}$$

$$= \sqrt{9t^4 - 24t^2 + 16 + 4t^2}$$

$$\text{speed} = \sqrt{9t^4 - 20t^2 + 16}$$

minimize speed^2

$$\text{Speed}^2 = 9t^4 - 20t^2 + 16$$

$$36t^3 - 40t = 0$$

$$t(36t^2 - 40) = 0$$

$$4t(9t^2 - 10) = 0$$

$$t = 0 \quad 9t^2 = 10$$

$$t^2 = \frac{10}{9}$$

$$t = \pm \frac{\sqrt{10}}{3}$$

use $t = \frac{\sqrt{10}}{3}$

$$\text{speed} = \sqrt{9\left(\frac{\sqrt{10}}{3}\right)^4 - 20\left(\frac{\sqrt{10}}{3}\right)^2 + 16}$$

$$= \sqrt{9\left(\frac{100}{81}\right) - 20\left(\frac{10}{9}\right) + 16}$$

$$= \sqrt{\frac{100}{9} - \frac{200}{9} + \frac{144}{9}} = \sqrt{\frac{44}{9}} = \boxed{\frac{2}{3}\sqrt{11}\ \text{cm/s}}$$