Homework 21 - Absolute and Conditional Convergence

1) $S_1 = 1$

$S_2 = 1 - \frac{1}{8} = 0.875$

$S_3 = 1 - \frac{1}{8} + \frac{1}{27} = 0.912$

$S_4 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} = 0.896$

$S_5 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} = 0.904$

$S_6 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} = 0.900$

$S_7 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} + \frac{1}{343} = 0.903$

$S_8 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} + \frac{1}{343} - \frac{1}{512} = 0.901$

$S_9 = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} + \frac{1}{343} - \frac{1}{512} + \frac{1}{729} = 0.902$

$S_{10} = 1 - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} + \frac{1}{343} - \frac{1}{512} + \frac{1}{729} - \frac{1}{1000} = 0.901$

$$|S - S_{10}| \leq \frac{1}{11^3}$$

$$-\frac{1}{11^3} \leq S - S_{10} \leq \frac{1}{11^3}$$

$$S_{10} - \frac{1}{11^3} \leq S \leq S_{10} + \frac{1}{11^3}$$

$$S_{10} - \frac{1}{11^3} \leq S \leq .901 + \frac{1}{11^3}$$

$$0.900 \leq S \leq 0.902$$

2) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720} + \frac{1}{5040} - \frac{1}{40320} + \frac{1}{362880} - \cdots$$

$$\frac{1}{362880} < 0.00005$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \approx 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720} + \frac{1}{5040} - \frac{1}{40320}$$

$$\approx 0.632118$$

3) $$\left|\frac{(-1)^{n+1}}{n^3}\right| = \frac{1}{n^3}$$

Find N such that $\frac{1}{(N+1)^3} \leq 10^{-5}$

$$(N+1)^3 \geq 10^5$$

$$N+1 \geq 46.4$$

$$N \geq 47.4$$

Use $N = 48$

4) a) $\sum_{n=2}^{\infty} \frac{\cos \pi n}{(\ln n)^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(\ln n)^2}$

$$a_n = \frac{(-1)^n}{(\ln n)^2}$$

$$b_n = |a_n| = \frac{1}{(\ln n)^2}$$

$$b_{n+1} < b_n \rightarrow \frac{1}{[\ln(n+1)]^2} < \frac{1}{(\ln n)^2} \text{ for all } n$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{(\ln n)^2} = 0$$

$\therefore \sum a_n$ is convergent by the AST (alternating series test)

b) $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1}$ This series is alternating, but it is divergent.

We can't use the AST to prove divergence.

$\lim_{n\to\infty} \frac{(-1)^n n^4}{n^3+1}$ does not exist

$$\lim_{n\to\infty} \frac{(-1)^n n^4}{n^3+1} \neq 0$$

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1}$ is divergent by the Divergence Test