Homework 26 - Taylor Series

1) $T(x) = f(3) + f'(3)(x-3) + \frac{f''(3)}{2!}(x-3)^2 + \frac{f'''(3)}{3!}(x-3)^3 + \dots$

$= 1 + 2(x-3) + \frac{12}{2}(x-3)^2 + \frac{3}{6}(x-3)^3 + \dots$

$= 1 + 2(x-3) + 6(x-3)^2 + \frac{1}{2}(x-3)^3 + \dots$

2) a) $f(x) = \frac{1}{1-2x}$

$T(x) = \sum_{n=0}^{\infty} (-2x)^n$ $\quad |-2x| < 1$

$= \sum_{n=0}^{\infty} (-2)^n x^n$ $\quad |x| < \frac{1}{2}$

b) $f(x) = \cos(3x)$

$T(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(3x)^{2n}}{(2n)!}$ $\quad (-\infty, \infty)$

c) $f(x) = \sin(x^3)$

$T(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{(2n+1)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{6n+3}$ $\quad (-\infty, \infty)$

d) $f(x) = x^2 e^{5x}$

$T(x) = x^2 \sum_{n=0}^{\infty} \frac{(5x)^n}{n!}$

$= \sum_{n=0}^{\infty} \frac{5^n}{n!} x^{n+2}$ $\quad (-\infty, \infty)$

e) $f(x) = x\ln(1-5x)$

$$T(x) = x\sum_{n=0}^{\infty} \frac{-(5x)^{n+1}}{n+1} \qquad -1 \le 5x < 1$$

$$= \sum_{n=0}^{\infty} \frac{-5^{n+1}}{n+1} x^{n+2} \qquad -\frac{1}{5} \le x < \frac{1}{5}$$

f) $f(x) = \cosh x = \frac{1}{2}e^{x} + \frac{1}{2}e^{-x}$

$$T(x) = \frac{1}{2}\sum_{n=0}^{\infty} \frac{x^n}{n!} + \frac{1}{2}\sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$$

$$= \frac{1}{2}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) + \frac{1}{2}\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)$$

$$= \frac{1}{2}\left(2 + 2\left(\frac{x^2}{2!}\right) + 2\left(\frac{x^4}{4!}\right) + \cdots\right)$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$T(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \qquad (-\infty, \infty)$$

3) a) $f(x)=\frac{1}{x}$, $c=1$

n	$f^{(n)}(x)$	$f^{(n)}(c)$	b_n
0	$\frac{1}{x}$	1	1
1	$-\frac{1}{x^2}$	-1	-1
2	$\frac{2}{x^3}$	2	1
3	$\frac{-3\cdot 2}{x^4}$	$-3\cdot 2$	-1
⋮			

$b_n=(-1)^n$

$$T(x)=\sum_{n=0}^{\infty}(-1)^n(x-1)^n$$

$0<x<2$

geometric
Convergent for $|-(x-1)|<1$

$|x-1|<1$

$-1<x-1<1$

$0<x<2$

b) $f(x)=x^4+3x-1$, $c=2$

n	$f^{(n)}(x)$	$f^{(n)}(2)$	b_n
0	x^4+3x-1	21	21
1	$4x^3+3$	35	35
2	$12x^2$	48	24
3	$24x$	48	8
4	24	24	1
5	0	0	0
⋮			

$T(x)=21+35(x-2)+24(x-2)^2+8(x-2)^3+(x-2)^4$

$(-\infty,\infty)$