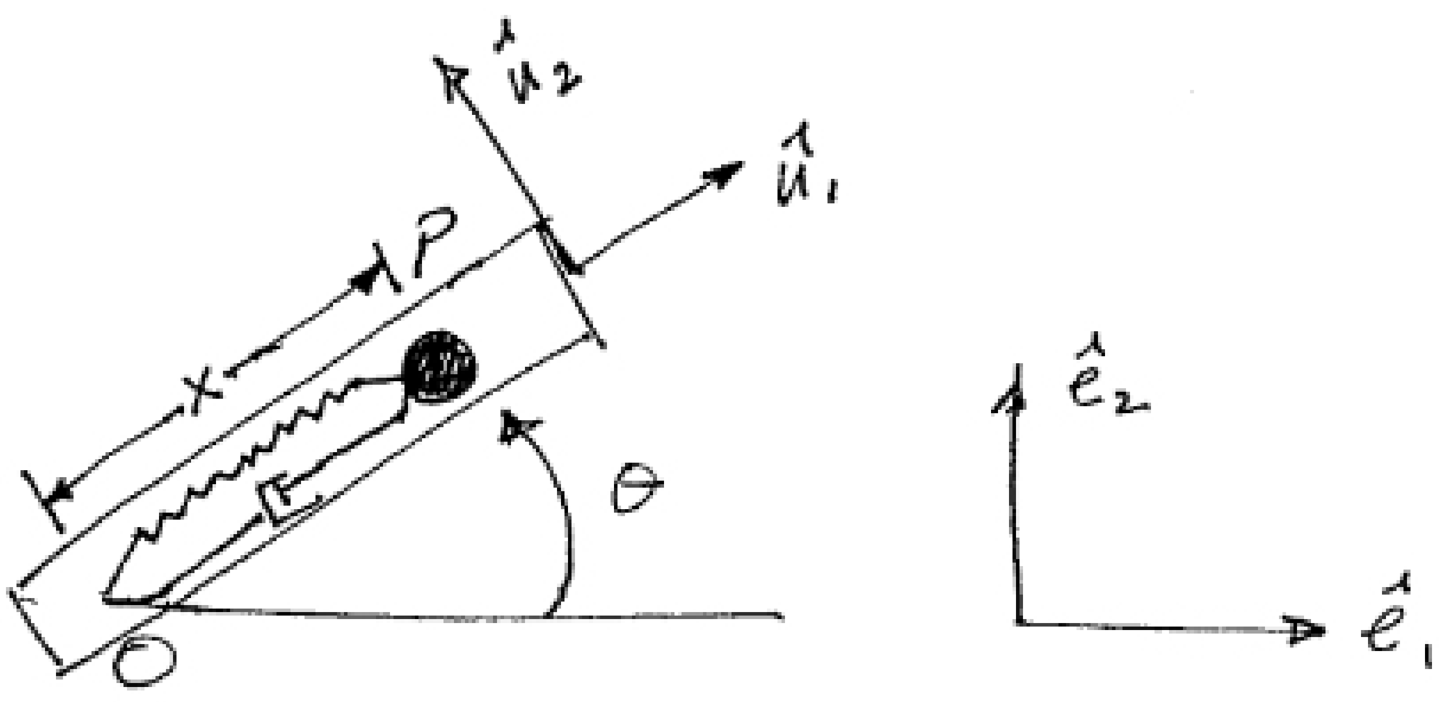
8/30/2012

AAE 340 HOMEWORK #2 SOLUTIONS

BRITTANY ESSINK
Updated by:
Paul Witsberger
8/29/2017

1. MASS-SPRING-DAMPER IN A SPINNING TUBE

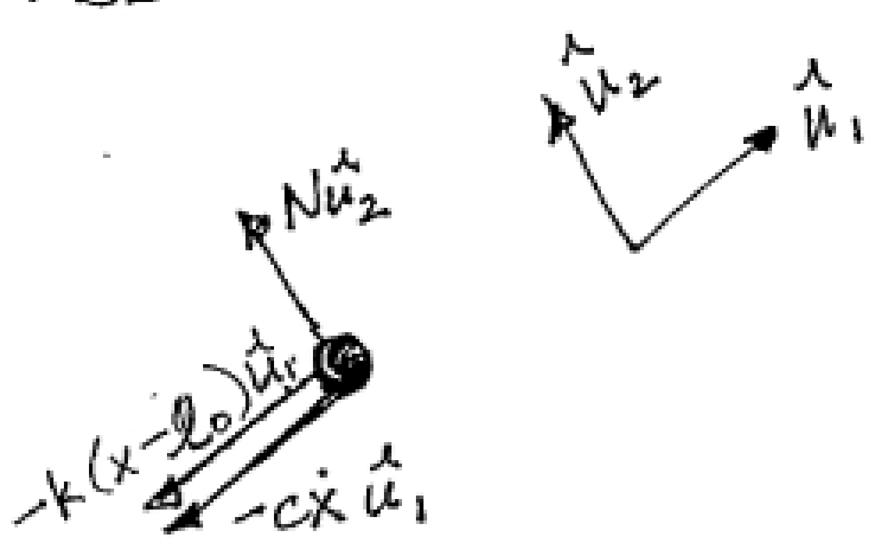


A PARTICLE, P, OF MASS m SLIDES IN A SMOOTH TUBE WHICH ROTATES AT A CONSTANT LENGTH, l_0, IS ZERO. THE SPRING HAS A CONSTANT OF k AND THE DAMPER HAS A DAMPING COEFFICIENT OF c.

ASSUMPTIONS:

- GRAVITY CAN BE NEGLECTED
- FRICTIONLESS

1a) STEP 1: FBD



STEP 2: FIND ${}^{e}\vec{a}^{OP}$ FROM KINEMATICS IN SAME COORDINATES AS FORCES (u-COORDINATES) WHERE ${}^{e}\vec{\omega}^{u} = \dot{\theta}\hat{u}_3$

$$\vec{R}^{OP} = x\hat{u}_1$$

VELOCITY:

$$ {}^{e}\vec{v}^{OP} = {}^{u}\frac{d}{dt}\vec{R}^{OP} + {}^{e}\vec{\omega}^{u} \times \vec{R}^{OP}$$

$${}^{e}\vec{V}^{OP} = {}^{u}\frac{d}{dt}(x\hat{u}_1) + (\dot{\theta}\hat{u}_3) \times (x\hat{u}_1)$$

$${}^{e}\vec{V}^{OP} = \dot{x}\hat{u}_1 + x\dot{\theta}\hat{u}_2$$

ACCELERATION:

$${}^{e}\vec{a}^{OP} = {}^{u}\frac{d}{dt}(\dot{x}\hat{u}_1 + x\dot{\theta}\hat{u}_2) + \dot{\theta}\hat{u}_3 \times (\dot{x}\hat{u}_1 + x\dot{\theta}\hat{u}_2)$$

$${}^{e}\vec{a}^{OP} = \ddot{x}\hat{u}_1 + \dot{x}\dot{\theta}\hat{u}_2 + \dot{x}\dot{\theta}\hat{u}_2 - x\dot{\theta}^2\hat{u}_1$$

$${}^{e}\vec{a}^{OP} = (\ddot{x} - x\dot{\theta}^2)\hat{u}_1 + 2\dot{x}\dot{\theta}\hat{u}_2$$

STEP 3: NEWTON'S 2ND LAW $(\sum \vec{F} = m\,{}^{e}\vec{a}^{OP})$

$$-k(x - l_0)\hat{u}_1 - c\dot{x}\hat{u}_1 + N\hat{u}_2 = m\left[(\ddot{x} - x\dot{\theta}^2)\hat{u}_1 + 2\dot{x}\dot{\theta}\hat{u}_2\right]$$

STEP 4: WRITE SCALAR EQUATIONS AND IDENTIFY EOM

$\hat{u}_1$:

$$-kx + kl_0 - c\dot{x} = m\ddot{x} - mx\dot{\theta}^2$$

$$m\ddot{x} + c\dot{x} - mx\dot{\theta}^2 + kx = kl_0$$

SINCE $l_0 = 0$

$$\ddot{x} + \frac{c}{m}\dot{x} - x\dot{\theta}^2 + \frac{k}{m}x = 0$$

$$\boxed{\ddot{x} + \frac{c}{m}\dot{x} + \left(\frac{k}{m} - \dot{\theta}^2\right)x = 0}$$

(1b) FIND AN EXPRESSION FOR THE CONSTRAINT FORCE ALONG $\hat{u}_2$

$\hat{u}_2$: $\boxed{N = 2\dot{x}\dot{\theta}m}$

1c) LET $k = m\left(\frac{d\theta}{dt}\right)^2$ AND SHOW THAT THE EOM CAN BE PUT IN THE FORM $\ddot{x} + 2\zeta\omega_n\dot{x} = 0$

$$\ddot{x} + \frac{c}{m}\dot{x} + \left(\frac{k}{m} - \dot{\theta}^2\right)x = 0$$

$k = m\dot{\theta}^2$

SUBSTITUTE IN

$$\ddot{x} + \frac{c}{m}\dot{x} + \left(\frac{\cancel{m}\dot{\theta}^2}{\cancel{m}} - \dot{\theta}^2\right)x = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \cancel{\left(\dot{\theta}^2 - \dot{\theta}^2\right)}x = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} = 0$$

LET $\frac{c}{m} = 2\zeta\omega_n$

$$\boxed{\ddot{x} + 2\zeta\omega_n\dot{x} = 0}$$

1d) FIND ANALYTICAL SOLUTION FOR $x(t)$ FOR 1c

$x = Ce^{st}$

$\dot{x} = Cse^{st}$

$\ddot{x} = Cs^2e^{st}$

$Cs^2e^{st} + 2\zeta\omega_n Cse^{st} = 0$

$\underbrace{Cse^{st}}_{s=0}\underbrace{\left[s + 2\zeta\omega_n\right]}_{s=-2\zeta\omega_n} = 0$

$x(t) = C_1 + C_2e^{-2\zeta\omega_n t}$

INITIAL CONDITIONS: $x(0) = 0, \dot{x}(0) = 0$

$x(0) = C_1 + C_2e^{-2\zeta\omega_n(0)}$

$x(0) = C_1 + C_2 = X_0$