Spring 2010 CSE310 Midterm Examination 02A (in class)

Instructions:

- There are five questions in this paper. Please use the space provided (below the questions) to write the answers.
- Budget your time to answer various questions and avoid spending too much time on a particular question.
- This is a **closed book** examination. You may not consult your books/notes.

NAME	
ASUID	

Problems	Score
P1	
P2	
P3	
P4	
P5	
Total	

P1. (20 points)

(10 pts) **This problem is related to order statistics.** What is the minimum number of element-wise comparisons needed to find the smallest element **and** the largest element in an un-ordered sequence of n elements? **Do not** use asymptotic notation in your answer.

Solution:

Let $f(n)$ denote the number of comparisons needed to find both the smallest and the largest elements in an array of n elements. First consider the case where n is even. Let $n = 2k$. For $k = 1$, we only need one comparison. Therefore $f(2) = 1$. Suppose we have computed the largest and the smallest elements among the first $2(k-1)$ elements, using $f(2k-2)$ comparisons. For the next two elements, we need 3 comparisons. Therefore $f(2k) = f(2k-2) + 3$. This leads to $\mathbf{f(2k) = 3k - 2}$ for $k = 1, 2, \ldots$.

Now consider the case where n is odd. Let $n = 2k + 1$. For $k = 0$, no comparison is needed, i.e., $f(1) = 0$. For $k = 1, 2, \ldots$, $f(2k+1) = f(2k) + 2$, because the last element needs to be compared with both the candidate for largest and the candidate for smallest. Therefore $\mathbf{f(2k+1) = 3k}$ for $k = 0, 1, 2, \ldots$. Combining both cases, we have $\mathbf{f(n) = \lceil \frac{3}{2} n \rceil - 2}$.

Grading Keys:

5 pts for correct answer;
4 pts for the answer of the form $\frac{3n}{2} + const$;
3 pts for the answer $\frac{3n}{2}$;
1 pts for the answer $2n - 3$.

Use the sequence

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8, 4, 7, 3, 6, 2, 5, 1
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to illustrate the process for finding the smallest element and the largest element. Show all the element-wise comparisons made for this example, in the correct order.

Solution:

The sequence of element-wise comparisons are: (8, 4), (7, 3), (8, 7), (3, 4), (6, 2), (8, 6), (3, 2), (5, 1), (8, 5), (1, 2).

Grading Keys:

+0.5 pts for each correct comparison.

(10 pts) **This problem is related to the linear time selection algorithm.** Assume that we are using the linear time selection algorithm with group size 5. Suppose that we have 25 numbers and that they have been grouped into 5 groups of size 5 (separated by ;). $8, 9, 10, 11, 12;\ 22, 21, 20, 19, 18;\ 28, 29, 30, 31, 32;\ 40, 38, 39, 41, 42;\ 50, 48, 49, 51, 52.$ What is the median of medians?

Solution:

The median of medians is 30.

Grading Keys:

5 pts for correct answer.

When we used groups of size 5, we have obtained the recurrence

$$T_5(n) \leq T_5(\lceil \frac{n}{5} \rceil) + T_5(\frac{7n}{10} + 6) + b \times n$$

for some constant b. What is the recurrence formula of the running time $T_9(n)$ for the corresponding select-k algorithm if we use groups of size 9.

Solutions:

An important part of the derivation is to derive a lower bound of the *upper left portion* of the elements. There are at least $\lceil \frac{\lceil \frac{n}{9} \rceil}{2} \rceil - 2$ columns, each with at least 5 elements that are smaller than the median of medians. Therefore the *upper left portion* contains at least $\frac{5n}{18} - 10$ elements that are smaller than the median of medians. This leads to the recurrence

$$T_9(n) \leq T_9(\lceil \frac{n}{9} \rceil) + T_9(\frac{13}{18}n + 10) + bn.$$

Grading Keys:

5 pts for correct answer;

2 pts for including either the first or the second term of the recurrence;

1 pts will be cut off for incorrect const in the second term of the recurrence.