NAME: Solutions

NetID:

MATH 285 E1/F1 Exam 1 (A) **September 19, 2014** **Instructor: Pascaleff**

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

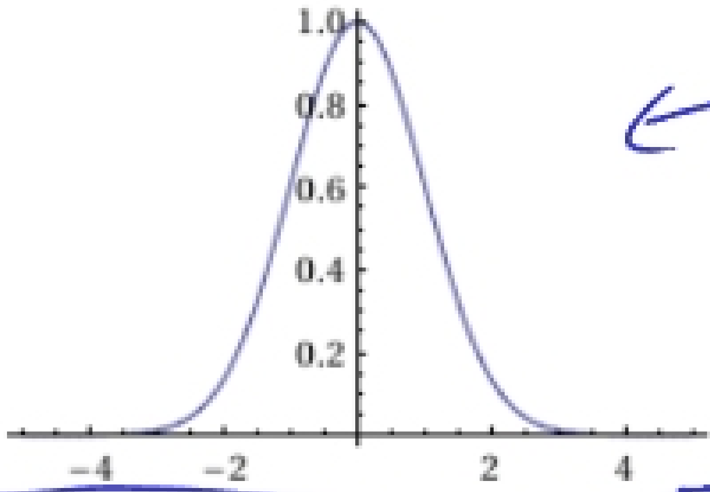
INSTRUCTIONS:

- Do all work on these sheets.
- Show all work.

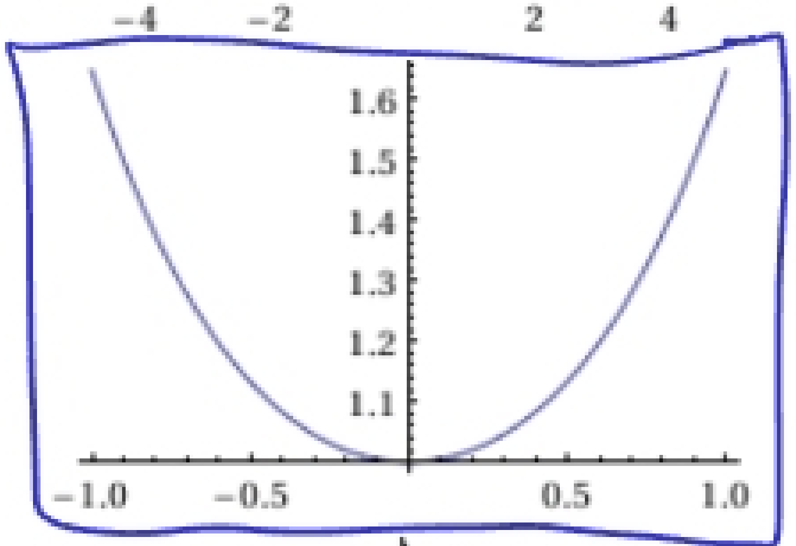
1. (20 points) Consider the differential equation

$$\frac{dy}{dx} = xy$$

Which of the following graphs could be a solution curve of this equation? Circle all that apply.



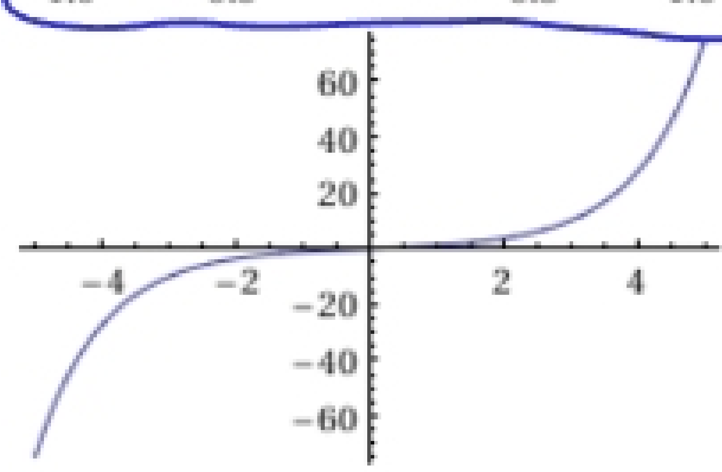
No, since slope in Quadrant I is negative, whereas $xy > 0$



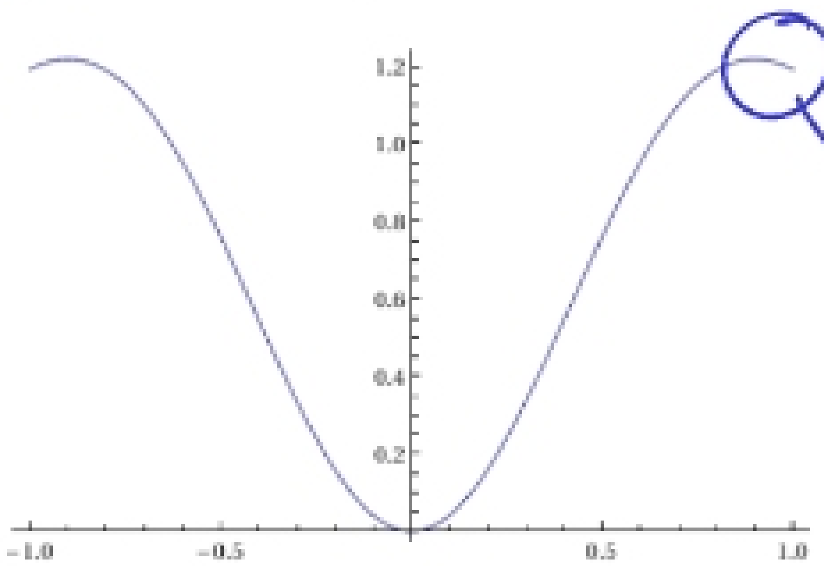
Yes, in fact $y(x) = Ce^{\frac{1}{2}x^2}$ is the general solution.

$\left[\frac{dy}{dx} - xy = 0 \quad \rho = e^{\int -x\,dx} = e^{-\frac{1}{2}x^2} \right.$

$\left. \frac{d}{dx}\left(e^{-\frac{1}{2}x^2} y\right) = 0 \quad e^{-\frac{1}{2}x^2} y = C \right]$



No, since $y \equiv 0$ is a solution, other solutions can't cross x-axis



No, slope is negative here, whereas $xy > 0$ there

2. (20 points) An object moves along a one-dimensional axis. Its motion is descibed by a function $x(t)$. It is subjected to an acceleration given by

$$a(t) = 1 + \pi \sin(\pi t).$$

Suppose that at $t = 0$, the velocity is zero: $v(0) = 0$. What is the net change in position between $t = 0$ and $t = 1$? That is, what is $x(1) - x(0)$?

$$\frac{dv}{dt} = a(t) = 1 + \pi \sin(\pi t)$$

$$v = \int a(t)\,dt = \int (1 + \pi \sin \pi t)\,dt = t - \cos(\pi t) + C$$

Find C:

$$0 = v(0) = 0 - \cos(\pi 0) + C = 0 - 1 + C$$

$$1 = C$$

So $v(t) = t - \cos(\pi t) + 1$

Integrate $v(t)$ from 0 to 1:

$$x(1) - x(0) = \int_0^1 (t - \cos(\pi t) + 1)\,dt$$

$$= \left[\frac{1}{2}t^2 - \frac{\sin(\pi t)}{\pi} + t\right]_0^1$$

$$= \frac{1}{2} - \frac{\sin(\pi)}{\pi} + 1 - 0 + \frac{\sin(0)}{\pi} - 0$$

$$= \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$$