NAME: Solutions

NetID:

MATH 285 E1/F1 Exam 1 (B) **September 19, 2014** **Instructor: Pascaleff**

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

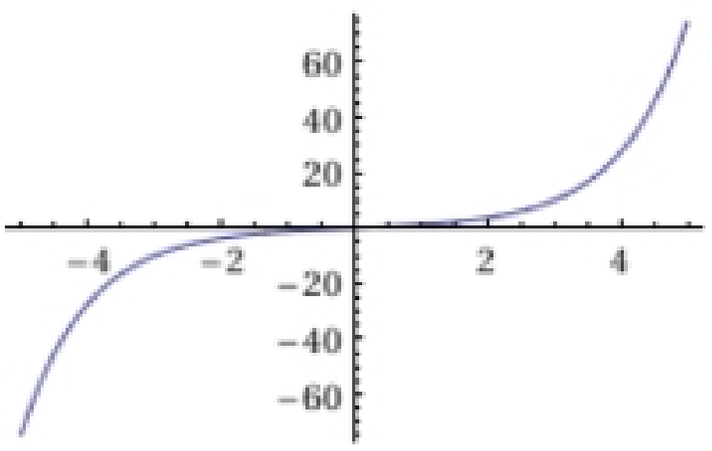
INSTRUCTIONS:

- Do all work on these sheets.
- Show all work.

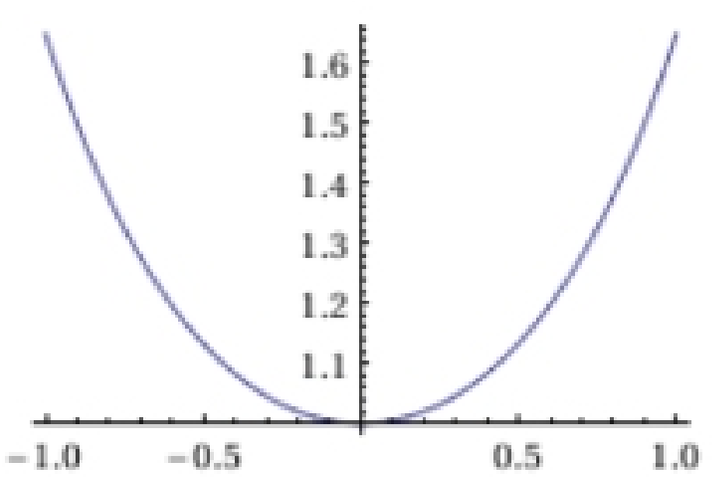
1. (20 points) Consider the differential equation

$$\frac{dy}{dx} = -xy$$

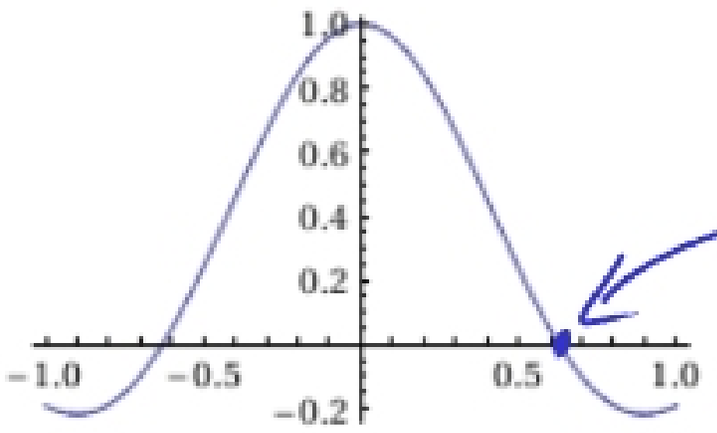
Which of the following graphs could be a solution curve of this equation? Circle all that apply.



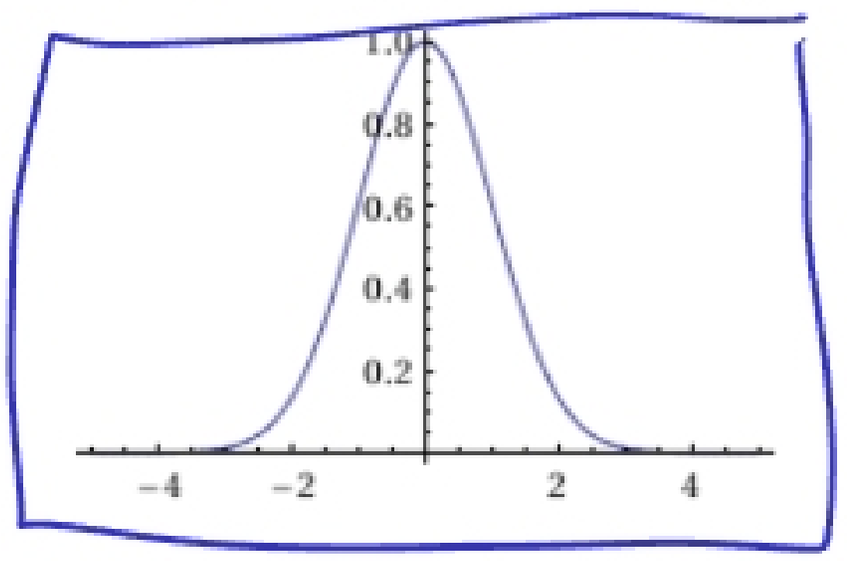
No, slope should be negative in Quadrant I



No slope is positive in Quadrant I whereas $-xy < 0$.



No, since slope at $y=0$ should be zero.



Yes. In fact general solution is $y = Ce^{-\frac{1}{2}x^2}$

$\frac{dy}{dx} + xy = 0 \quad \rho(x) = e^{\int x\,dx} = e^{\frac{1}{2}x^2}$

$\frac{d}{dx}(e^{\frac{1}{2}x^2} y) = 0 \rightarrow e^{\frac{1}{2}x^2} y = C.$

2. (20 points) An object moves along a one-dimensional axis. Its motion is descibed by a function $x(t)$. It is subjected to an acceleration given by

$$a(t) = 2 + 2\pi \sin(\pi t).$$

Suppose that at $t = 0$, the velocity is zero: $v(0) = 0$. What is the net change in position between $t = 0$ and $t = 1$? That is, what is $x(1) - x(0)$?

$\frac{dv}{dt} = a(t) = 2 + 2\pi \sin(\pi t)$

$v = \int a(t)\,dt = \int (2 + 2\pi \sin \pi t)\,dt = 2t - 2\cos(\pi t) + C$

Find C:

$0 = v(0) = 0 - 2\cos(\pi 0) + C = 0 - 2 + C$

$2 = C$

So $v(t) = 2t - 2\cos(\pi t) + 2$

Integrate $v(t)$ from 0 to 1:

$$x(1) - x(0) = \int_0^1 (2t - 2\cos(\pi t) + 2)\,dt$$

$$= \left[t^2 - 2\,\frac{\sin(\pi t)}{\pi} + 2t\right]_0^1$$

$$= 1 - 2\,\frac{\sin(\pi)}{\pi} + 2 - 0 + 2\,\frac{\sin(0)}{\pi} - 0$$

$= 1 + 2 = \boxed{3}$