

NAME: *Solutions*

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MATH 285 E1/F1 Exam 2 (C)

October 17, 2014

Instructor: Pascaleff

**INSTRUCTIONS:**

- Do all work on these sheets.
- Show all work.
- The exam is 50 minutes.
- Do not discuss this exam with anyone until after 3:00 pm on Oct. 17, 2014.

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) Let  $P(t)$  denote the population of a penguin colony in Antarctica. We assume that each female lays one egg each year, so the birth rate is 0.5 births per penguin per year. Due to scarce resources, the death rate depends on the population as  $.03 + .01P$  deaths per penguin per year.

Suppose that the penguin population is in *equilibrium*, meaning that it is constant in time:  $P(t) = P_0$ . What are the possible values of the equilibrium (constant) population  $P_0$ ?

$$\frac{dP}{dt} = .5P - (.03 + .01P)P$$

$$\frac{dP}{dt} = .47P - .01P^2$$

If population is constant,  $P(t) = P_0$ , then  $\frac{dP}{dt} = 0$

$$0 = \frac{dP}{dt} = .47P_0 - .01P_0^2$$

$$0 = P_0(.47 - .01P_0)$$

$$\Rightarrow P_0 = 0 \quad \text{or} \quad .47 - .01P_0 = 0$$

$$.47 = .01P_0$$

$$47 = P_0$$

$$\text{So } \boxed{P_0 = 0 \quad \text{or} \quad P_0 = 47}$$

2. (20 points) Show that the functions  $f(x) = e^{5x}$  and  $g(x) = e^{x^2+1}$  and  $h(x) = 4e^{x^2}$  are *not* linearly independent. That is, find constants  $A, B, C$  such that

$$Af(x) + Bg(x) + Ch(x) = 0 \text{ for all } x$$

$$g(x) = e^{x^2+1} = e^{x^2} e^1$$

$$\text{So } 4g(x) - eh(x) = 4e e^{x^2} - e 4 e^{x^2} = 0$$

$$A = 0, B = 4, C = -e$$

$$0f(x) + 4g(x) - eh(x) = 0$$