

EE 503 : Problem Set #1 Solutions

1-1 a) Sample Space

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

b) A corresponds to the event that the number of dots in the first toss is not less than the number of dots in the second toss..

$$A = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), \\ (5, 2), (5, 3), (5, 4), (5, 5), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

c) B corresponds to the event that the number of dots in the first toss is 6.

$$B = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

d) B is a subset of A. $B \subset A$. Hence, B implies A.

e)

$$B^c = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$$

$$A \cap B^c = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3), \\ (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)\}$$

Thus, $A \cap B^c$ corresponds to the event that the numbers of dots in the first toss are not less than the number of dots in the second and the first toss is not 6.

f) C corresponds to the event that the number of dots on the dice differ by 2.

$$C = \{(1, 3), (2, 4), (3, 1), (3, 5), (4, 2), (4, 6), (5, 3), (6, 4)\} \\ \rightarrow A \cap C = \{(3, 1), (4, 2), (5, 3), (6, 4)\}$$

1-2 a) Sample Space

$$S = \{0, 1, 2, 3, 4, 5\}$$

b) A corresponds to the event that the magnitude of the difference is 3.

$$A = \{3\}$$

c)

$$\{\text{difference} = 0\} = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$\{\text{difference} = 1\} = \{(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)\}$$

$$\{\text{difference} = 2\} = \{(1, 3), (3, 1), (2, 4), (4, 2), (3, 5), (5, 3), (4, 6), (6, 4)\}$$

$$\{\text{difference} = 3\} = \{(1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3)\}$$

$$\{\text{difference} = 4\} = \{(1, 5), (5, 1), (2, 6), (6, 2)\}$$

$$\{\text{difference} = 5\} = \{(1, 6), (6, 1)\}$$

1-3 a)

$$\begin{aligned} (A \cup B) \cap (A^c \cup B^c) &= (A \cap (A^c \cup B^c)) \cup (B \cap (A^c \cup B^c)) \\ &= (A \cap A^c) \cup (A \cap B^c) \cup (B \cap A^c) \cup (B \cap B^c) \\ &= (A \cap B^c) \cup (A^c \cap B) \end{aligned}$$

Since $(A \cap A^c) = (B \cap B^c) = \emptyset$.

b)

$$\begin{aligned} (A \cap B \cap C) - A^c \cup B^c \cup C^c &= (A^c \cap (B \cup B^c) \cap (C \cup C^c)) \cup (B^c \cap (A \cup A^c) \cap (C \cup C^c)) \\ &\quad \cup (C^c \cap (A \cup A^c) \cap (C \cup C^c)) \\ &= (A^c \cap B \cap C) \cup (A \cap B^c \cap C) \cup (A \cap B \cap C^c) \\ &\quad \cup (A^c \cap B^c \cap C) \cup (A^c \cap B \cap C^c) \cup (A \cap B^c \cap C^c) \cup (A^c \cap B^c \cap C^c) \end{aligned}$$

1-4

$$A = \{2 \leq x \leq 5\}$$

$$B = \{3 \leq x \leq 6\}$$

$$A \cup B = \{2 \leq x \leq 6\}$$

$$A \cap B = \{3 \leq x \leq 5\}$$

$$(A \cap B)^c = \{x < 3\} \cup \{5 < x\}$$

$$(A \cup B) \cap (A \cap B)^c = \{2 \leq x < 3\} \cup \{5 < x \leq 6\}$$

1-5 a)

$$(A^c \cup B^c)^c \cup (A^c \cap B)^c = (A \cap B) \cup (A \cap B^c) \quad (\text{de Morgan's Law}) \quad (1)$$

$$= A \cap (B \cup B^c) \quad (2)$$

$$= A \quad (3)$$

b)

$$\begin{aligned}(A \cup B) \cap (A \cap B)^c &= (A \cup B) \cap (A^c \cup B^c) \\ &= (A \cap A^c) \cup (A \cap B^c) \cup (B \cap A^c) \cup (B \cap B^c) \\ &= (A \cap B^c) \cup (B \cap A^c)\end{aligned}$$