EE 503 : Problem Set #3 Solutions

2-46 a) If toppings can be repeated, it is a problem of sampling with replacement but without ordering. So the number of combinations is

$$\binom{15-1+4}{4} = 3060$$

b) If toppings cannot be repeated, it is equivalent to sampling without replacement and ordering. So the number of combinations is

$$\binom{15}{4} = 1365$$

2-53 a) For one condiment, the number of combinations is $\binom{6}{1} = 6$.

b) For two condiments, the number of combinations is $\binom{6}{2} = 15$.

c) For none, some, or all of the condiments, the number of combinations is

$$\sum_{k=0}^{6} \binom{6}{k} = 2^6 = 64.$$

2-80 Let $E = \{\text{selected chip is defective}\}$, $A = \{\text{selected chip is from } A\}$, $B = \{\text{selected chip is from } B\}$, and $C = \{\text{selected chip is from } C\}$. Then the probability of the selected chip is defective is given by

$$\begin{aligned} P(E) &= P(E(A+B+C)) \\ &= P(EA) + P(EB) + P(EC) \\ &= P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C) \\ &= 0.005 \cdot 0.5 + 0.001 \cdot 0.1 + 0.01 \cdot 0.4 \\ &= 0.0066 \end{aligned}$$

Then the probability that the chip's manufacturer is A given the chip is defective is given by

$$\begin{aligned} P(A|E) &= \frac{P(E|A)P(A)}{P(E)} \\ &= \frac{0.005 \cdot 0.5}{0.0066} \\ &\approx 0.3788 \end{aligned}$$

And the probability that the chip's manufacturer is C given the chip is defective is given by

$$\begin{aligned} P(C|E) &= \frac{P(E|C)P(C)}{P(E)} \\ &= \frac{0.01 \cdot 0.4}{0.0066} \\ &\approx 0.6061 \end{aligned}$$

2-97 a) Let N be the random variable of the number of errors in a block. Then

$$P(N = 1) = \binom{100}{1} p(1-p)^{99} = 0.3697$$

and

$$P(N = 0) = (1-p)^{100} = 0.3660$$

So

$$P(N \le 1) = P(N = 0) + P(N = 1) = 0.3697 + 0.3660 = 0.7357$$

b) This problem statement seems to be ambiguous since "M retransmissions are required" may imply two cases. First of all, there are M retransmissions and the transmission is successful on the last one. Second, there are at least M retransmissions. Both the answers are correct.

We first consider the first case. The probability that a block has more than 1 error is given by

$$P(N > 1) = 1 - P(N \le 1) = 0.2643$$

Let W be the event that the first M transmissions are not successful and the $(M+1)$-th transmission is successful. Then

$$\begin{aligned} P(\text{exactly } M \text{ retransmissions are required}) &= P(W) \\ &= P(N > 1)^M P(N \le 1) \\ &= 0.2643^M \cdot 0.7357 \end{aligned}$$

For the second case, we can write

$$\begin{aligned} &P(\text{at least } M \text{ retransmissions required}) \\ =&P(\text{exactly } M \text{ retransmissions required}) + P(\text{exactly } M+1 \text{ retransmissions required}) + \cdots \\ =&P(N > 1)^M P(N \le 1) + P(N > 1)^{M+1} P(N \le 1) + \cdots \\ =&\frac{P(N > 1)^M}{1 - P(N > 1)} P(N \le 1) \\ =&P(N > 1)^M \\ =&0.2643^M \end{aligned}$$