Name (PRINTED): ______________________

Student ID #: ______________________

Section # (or TA's: name and time) ______________________

CMSC 250 Quiz #6 ANSWERS Wednesday, Mar. 3, 2004

Write all answers legibly in the space provided. The number of points possible for each question is indicated in square brackets – the total number of points on the quiz is 30, and you will have exactly 20 minutes to complete this quiz. You may not use calculators, textbooks or any other aids during this quiz.

1. [24 pnts.] Disprove by counter example or Prove each of the following:

   a. The product of any rational number with an integer is a rational number.

      $\forall x \in Q, \forall y \in Z, xy \in Q$
      PROOF:
      Let x by arbitrary in Q and let y be arbitrary in Z.
      Since x is rational, $\exists a, b \in Z,\ x = \frac{a}{b}$ where $b \neq 0$ by definition of rational
      The product of x and y can be written as $y * \frac{a}{b}$ by substitution
      After multiplying, $xy = \frac{ya}{b}$
      Since $ya \in Z$ by closure of Z in multiplication
      and $b \in Z$ and $b \neq 0$ because it was defined as such above.
      Therefore $xy \in Q$ by definition of rational. $\forall x \in Q, \forall y \in Z, xy \in Q$ by generalizing from the generic particula

   b. For all integers n, if n is odd then n^2 is odd.

      $\forall n \in Z, n \in Z^{odd} \rightarrow n^2 \in Z^{odd}$ PROOF: Let n be arbitrary in Z. |Assume $n \in Z^{odd}$
      |Since n is an odd integer, $\exists a \in Z\ n = 2a + 1$ by the definition of odd.
      |Since $n = 2a + 1$, $n^2 = (2a + 1)^2$ by squaring both sides.
      |$(2a + 1)^2 = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1$ by algebra.
      |Since $2a^2 + 2a$ is an integer by closure of integers during addition and multiplication, n^2 is also even by the definition of odd.
      $n \in Z^{odd} \rightarrow n^2 \in Z^{odd}$ by closing the conditional world
      $\forall n \in Z, n \in Z^{odd} \rightarrow n^2 \in Z^{odd}$ by Generalizing from the Generic Particular

   c. For all integers n and m, $(n + m) > m$.

      False $n = -2$ and $m = 5$, $(n + m) = -2 + 5 = 3$ which is not greater than 5.

2. [6 pnts.] State Yes or No for each of the following (only a small justification is needed for your answer not a complete proof). Assume a, b and c are integers and x, y and z are rationals for all of the following questions.

a. ___________NO___________If a, b, and c are even, $\frac{a+b+c}{2}$ is also even.

justification: a = 2, b = 2 and c = 2 then a+b+c = 6 and 6/2 is 3 which is odd.

b. ___________NO___________$x + y \leq x \cdot y$

justification: x = 2 and y = -2, x+y = 0 and xy = -4, but $0 \not\leq -4$

c. ___________NO___________If $a > b$ and $x > y$, then $a \cdot x > b \cdot y$ justification: a = 1 and b = -2 and x = -4 and y = -5, while it is true that 1 ¿ -2 and -4 ¿ -5, it is not true that -4 ¿ 10