

Answers to Selected Exercises

Chapter 1

- 1.1 1. (a) (11, 3) (b) (4, 1, 3) (c) (-2, 0, 3, 1)
(d) (-2, 3, 0, 3, 1)

2. (a) $\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

3. (a) One solution. The two lines intersect at the point (3, 1).
(b) No solution. The lines are parallel.
(c) Infinitely many solutions. Both equations represent the same line.
(d) No solution. Each pair of lines intersect in a point, however, there is no point that is on all three lines.

4. (a) $\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array} \right]$ (c) $\left[\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & -6 \end{array} \right]$

(d) $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 3 & 3 \end{array} \right]$

6. (a) (1, -2) (b) (3, 2) (c) $(\frac{1}{2}, \frac{2}{3})$
(d) (1, 1, 2) (e) (-3, 1, 2)
(f) (-1, 1, 1) (g) (1, 1, -1)
(h) (4, -3, 1, 2)
7. (a) (2, -1) (b) (-2, 3)
8. (a) (-1, 2, 1) (b) (3, 1, -2)

- 1.2 1. Row echelon form: (a), (c), (d), (g), and (h)
Reduced row echelon form: (c), (d), and (g)

2. (a) Inconsistent
(c) consistent, infinitely many solutions
(d) consistent (4, 5, 2)
(e) inconsistent
(f) consistent, (5, 3, 2)

3. (b) β
(c) $\{(2 + 3\alpha, \alpha, -2) \mid \alpha \text{ real}\}$
(d) $\{(5 - 2\alpha - \beta, \alpha, 4 - 3\beta, \beta) \mid \alpha, \beta \text{ real}\}$
(e) $\{(3 - 5\alpha + 2\beta, \alpha, \beta, 6) \mid \alpha, \beta \text{ real}\}$
(f) $\{(\alpha, 2, -1) \mid \alpha \text{ real}\}$

4. (a) x_1, x_2, x_3 are lead variables.
(c) x_1, x_3 are lead variables and x_2 is a free variable.
(e) x_1, x_4 are lead variables and x_2, x_3 are free variables.

5. (a) (5, 1) (b) inconsistent (c) (0, 0)
(d) $\left\{ \left(\frac{5 - \alpha}{4}, \frac{1 + 7\alpha}{8}, \alpha \right) \mid \alpha \text{ real} \right\}$
(e) $\{(8 - 2\alpha, \alpha - 5, \alpha)\}$
(f) inconsistent

- (g) inconsistent (h) inconsistent
(i) $(0, \frac{2}{3}, 1)$

- (j) $\{(2 - 6\alpha, 4 + \alpha, 3 - \alpha, \alpha)\}$
(k) $\{(\frac{15}{4} - \frac{3}{8}\alpha - \beta, -\frac{1}{4} - \frac{1}{8}\alpha, \alpha, \beta)\}$

6. (a) (0, -1)
(b) $\{(\frac{3}{4} - \frac{5}{8}\alpha, -\frac{1}{4} - \frac{1}{8}\alpha, \alpha, 3) \mid \alpha \text{ is real}\}$
(d) $\{\alpha(-\frac{4}{3}, 0, \frac{1}{3}, 1)\}$

8. $a \neq -2$

9. $\beta = 2$

10. (a) $a = 5, b = 4$ (b) $a = 5, b \neq 4$

11. (a) (-2, 2) (b) (-7, 4)

12. (a) (-3, 2, 1) (b) (2, -2, 1)

15. $x_1 = 280, x_2 = 230, x_3 = 350, x_4 = 590$

19. $x_1 = 2, x_2 = 3, x_3 = 12, x_4 = 6$

20. 6 moles N_2 , 18 moles H_2 , 21 moles O_2

21. All three should be equal (i.e., $x_1 = x_2 = x_3$).

22. (a) (5, 3, -2) (b) (2, 4, 2)

(c) (2, 0, -2, -2, 0, 2)

1.3 1. (a) $\begin{bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 1 & 6 \\ -5 & 1 & 2 \\ 3 & -2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 & 2 \\ 5 & -3 & -1 \\ -4 & 16 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 16 \\ 2 & -1 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{bmatrix}$

(h) $\begin{bmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{bmatrix}$

2. (a) $\begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{bmatrix}$

(f) $\begin{bmatrix} 6 & 4 & 8 & 10 \\ -3 & -2 & -4 & -5 \\ 9 & 6 & 12 & 15 \end{bmatrix}$

(b) and (e) are not possible.

3. (a) 3×3 (b) 1×2

4. (a) $\begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$

9. (a) $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$

10. (a) inconsistent (b) consistent

(c) inconsistent

13. $\mathbf{b} = (8, -7, -1, 7)^T$

17. $b = a_{22} - \frac{a_{12}a_{21}}{a_{11}}$

1.4 7. $A = A^2 = A^3 = A^n$

8. $A^{2n} = I, A^{2n+1} = A$

13. (a) $\begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}$

31. 4500 married, 5500 single

32. (b) 0 walks of length 2 from V_2 to V_3 and 3 walks of length 2 from V_2 to V_5 (c) 6 walks of length 3 from V_2 to V_3 and 2 walks of length 3 from V_2 to V_5

33. (a) $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(c) 5 walks of length 3 from V_2 to V_4 and 7 walks of length 3 or less

1.5 1. (a) type I

(b) not an elementary matrix

(c) type III (d) type II

3. (a) $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

4. (a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. (a) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(b) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

6. (a) $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

(c) $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

8. (a) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$,

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

9. (b) (i) $(0, -1, 1)^T$, (ii) $(-4, -2, 5)^T$, (iii) $(0, 3, -2)^T$

10. (a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

- CHAPTER TEST A**
 1. False 2. True 3. True 4. True 5. False
 6. False 7. False 8. False 9. False 10. True
 11. True 12. True 13. True 14. False
 15. True

13. $A^2 = \begin{bmatrix} B & O \\ O & B \end{bmatrix}$, $A^4 = \begin{bmatrix} B^2 & O \\ O & B^2 \end{bmatrix}$
 14. (a) $\begin{bmatrix} O & I \\ I & O \end{bmatrix}$ (b) $\begin{bmatrix} -B & I \\ I & O \end{bmatrix}$

(d) $\begin{bmatrix} 3 & -3 \\ 2 & -2 \\ 1 & -1 \\ 5 & -5 \\ 4 & -4 \end{bmatrix}$

- 2.1** 1. (a) $\det(M_{21}) = -8$, $\det(M_{22}) = -2$, $\det(M_{23}) = 5$
 (b) $A_{21} = 8$, $A_{22} = -2$, $A_{23} = -5$
 2. (a) and (c) are nonsingular.
 3. (a) 1 (b) 4 (c) 0 (d) 58
 (e) -39 (f) 0 (g) 8 (h) 20
 4. (a) 2 (b) -4 (c) 0 (d) 0
 5. $-x^2 + ax^2 + bx + c$
 6. $\lambda = 6$ or -1
2.2 1. (a) -24 (b) 30 (c) -1
 2. (a) 10 (b) 20
 3. (a), (c), and (f) are singular while (b), (e), and (d) are nonsingular.
 4. $c = 5$ or -3
 7. (a) 20 (b) 108 (c) 160 (d) $\frac{2}{3}$
 9. (a) -6 (c) 6 (e) 1
 13. $\det(A) = \det(M_{22})$

- 2.3** 1. (a) $\det(A) = -7$, $\text{adj } A = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$.
 $A^{-1} = \begin{bmatrix} \frac{1}{-7} & \frac{2}{-7} \\ \frac{3}{-7} & \frac{-1}{-7} \end{bmatrix}$
 (c) $\det(A) = 3$, $\text{adj } A = \begin{bmatrix} -3 & 5 & 2 \\ 0 & 1 & 1 \\ 6 & -8 & -5 \end{bmatrix}$
 $A^{-1} = \frac{1}{3} \text{adj } A$

2. (a) $(\frac{7}{2}, \frac{1}{2})$ (b) $(\frac{11}{5}, -\frac{2}{5})$
 (c) $(4, -2, 2)$ (d) $(2, -1, 2)$
 (e) $(-\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, 0)$
 3. $-\frac{4}{3}$
 4. $(\frac{1}{2}, -\frac{2}{3}, 1)^T$

- 1.6** 1. (b) $\begin{bmatrix} I & A^{-1} \\ A^T A & A^T \end{bmatrix}$ (c) $\begin{bmatrix} A & I \\ I & A \end{bmatrix}$

12. (a) $\begin{bmatrix} 20 & -5 \\ -34 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -2 \\ -2 & 2 \end{bmatrix}$
 11. (a) $\begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -8 & 5 \\ -14 & 9 \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{1}{2} & -1 & -\frac{1}{2} \\ \frac{3}{2} & 1 & \frac{1}{2} \end{bmatrix}$

(e) $\begin{bmatrix} 2 & -3 & 3 \\ -\frac{3}{2} & \frac{3}{2} & -1 \\ -\frac{3}{2} & -\frac{3}{2} & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 3 & 0 & -5 \\ 0 & \frac{1}{2} & 0 \\ -1 & 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & 3 \\ \frac{5}{2} & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ -1 & \frac{1}{3} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$

3. (a) $AB_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, $AB_2 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

(d) $AA^T + I$ (e) $\begin{bmatrix} A & I \\ I & A^{-1} \end{bmatrix}$

(c) $AB = \begin{bmatrix} 3 & 4 \\ 3 & -1 \end{bmatrix}$

4. (a) $\begin{bmatrix} 3 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 \\ 3 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 \\ 3 & 1 & 1 & 1 \end{bmatrix}$

5. (b) $\begin{bmatrix} 0 & 2 & 0 & -2 \\ 8 & 5 & 8 & -5 \\ 3 & 2 & 3 & -2 \\ 5 & 3 & 5 & -3 \end{bmatrix}$