Solutions to the Second Set of Practice Problems

1. Write as a single fraction:

$$\frac{3}{x(x-3)} - \frac{2x}{(x-3)(x+2)} + \frac{1}{x}.$$

Solution.

$$\begin{aligned}
\frac{3}{x(x-3)} - \frac{2x}{(x-3)(x+2)} + \frac{1}{x} &= \frac{3(x+2)}{x(x-3)(x+2)} - \frac{2x(x)}{x(x-3)(x+2)} + \frac{(x+2)(x-3)}{x(x+2)(x-3)} \\
&= \frac{3(x+2) - 2x^2 + (x+2)(x-3)}{x(x-3)(x+2)} \\
&= \frac{3x + 6 - 2x^2 + x^2 - x - 6}{x(x-3)(x+2)} \\
&= \frac{-x^2 + 2x}{x(x-3)(x+2)} \\
&= \frac{x(-x+2)}{x(x-3)(x+2)} \\
&= \boxed{\frac{-x+2}{(x-3)(x+2)}}
\end{aligned}$$

□

2. Simplify

$$\frac{\frac{8x^3+x}{e^x}}{x\tan(x)\sqrt{4x}}.$$

Solution.

$$\begin{aligned}
\frac{\frac{8x^3+x}{e^x}}{x\tan(x)\sqrt{4x}} &= \frac{8x^3+x}{e^x}\frac{1}{x\tan(x)\sqrt{4x}} \\
&= \frac{x(8x^2+1)}{e^x x\tan(x)(2\sqrt{x})} \\
&= \boxed{\frac{8x^2+1}{e^x\tan(x)(2\sqrt{x})}}
\end{aligned}$$

□

3. Find all the solutions to $|x^2 - 3| = 1$.

Solution.

$$\begin{aligned} |x^2-3| &= 1 \\ x^2-3 &= \pm 1 \\ x^2 &= 4, 2 \end{aligned}$$

The solutions to the above equality are the solutions to $x^2 = 4$ and the solutions to $x^2 = 2$. The solutions to $x^2 = 4$ are $\boxed{x = 2}$ and $\boxed{x = -2}$. The solutions to $x^2 = 2$ are $\boxed{x = \sqrt{2}}$ and $\boxed{x = -\sqrt{2}}$. □

4. Find all the solutions to $(x^2-4)(x^2+5x+4) = 0$.

   Solution. The solutions to $(x^2-4)(x^2+5x+4) = 0$ are the solutions to $x^2 - 4 = 0$ and the solutions to $x^2 + 5x + 4 = 0$. The solutions of $x^2 - 4 = 0$ are $\boxed{x = 2}$ and $\boxed{x = -2}$. Similarly, the solutions of $x^2 + 5x + 4 = 0$ are $\boxed{x = -1}$ and $\boxed{x = -4}$ (use the quadratic formula if you don't see how to factor it otherwise). □

5. Find the equation of the line that passes through the points $(\pi, 0)$ and (0,5).

   Solution. $y - 0 = (\frac{5-0}{0-\pi})(x - \pi)$
   $\boxed{y = (-5/\pi)(x - \pi)}$

   □

6. Find the equation of the line that passes through the point (5,-1) and has slope e.

   Solution. $y - (-1) = e(x - 5)$
   $\boxed{y + 1 = e(x - 5)}$ □

7. Find the y-intercepts and x-intercepts of $f(x) = e^{x+2} - 2$.

   Solution. The y-intercept is $f(0) = e^{0+2} - 2 = \boxed{e^2 - 2}$. The x-intercepts are the solutions to the equation $0 = e^{x+2} - 2$.

$$\begin{aligned} e^{x+2} - 2 &= 0 \\ e^{x+2} &= 2 \\ \ln(e^{x+2}) &= \ln(2) \\ x + 2 &= \ln(2) \\ x &= \boxed{\ln(2) - 2} \end{aligned}$$

□

8. Find the y-intercepts and x-intercepts of $f(x) = (3 - x)(\ln(x) + 1)$.

Solution. The y-intercept is $f(0) = (3-0)(\ln(0)+1) = (3)(1+1) = \boxed{6}$. The x-intercepts are the solutions to the equation $0 = (3-x)(\ln(x)+1)$. The expression $3-x$ equals 0 when $\boxed{x=3}$. The expression $\ln(x)+1$ equals 0 when $\ln(x) = -1$ which happens when $x = e^{\ln(x)} = \boxed{e^{-1}}$. □

9. If $\tan\theta = 3/4$, find $\sin\theta$.

Solution. Draw a right triangle with an angle θ, the side opposite to θ with length 3, and the side adjacent to θ with length 4. (Recall that tan is "opposite over adjacent".) The hypotenuse must then have length $\sqrt{4^2+3^2} = \sqrt{25} = 5$. Thus $\boxed{\sin\theta = \frac{3}{5}}$. □

10. What is the value of $\sin(5\pi/6)$, $\cos(7\pi/4)$, $\csc(\pi/2)$ and $\cot(\pi/6)$?

Solution. Using the unit circle like we did in class on Friday, we can determine the following values:

$$\begin{aligned}
\sin(\frac{5\pi}{6}) &= \frac{1}{2} \\
\cos(\frac{7\pi}{4}) &= \frac{\sqrt{2}}{2} \\
\csc(\frac{\pi}{2}) &= \frac{1}{\sin(\pi/2)} \\
&= 1/1 \\
&= 1 \\
\cot(\frac{\pi}{6}) &= \frac{\cos(\pi/6)}{\sin(\pi/6)} \\
&= \frac{\sqrt{3}/2}{1/2} \\
&= \sqrt{3}
\end{aligned}$$

□

11. Find all the solutions to $\ln((2x^2-1)) = 0$.

Solution.

$$\begin{aligned}
\ln((2x^2-1)) &= 0 \\
e^{\ln((2x^2-1))} &= e^0 \\
2x^2-1 &= 1 \\
2x^2 &= 2 \\
x^2 &= 1 \\
x &= \boxed{\pm 1}.
\end{aligned}$$

□