Last topic for the semester will be solving exponential and logarithmic equations.

An exponential equation contains a^x or e^x where $a > 1$. To solve we will use several items from the sections in chapter 5. To solve the exponential we will need to isolate the exponential term. Take the logarithm of both sides and solve.

Example 1: Solve for x: $3^x = 5$

$x = \log_3 5$

Example 2: Solve for x: $8e^{2x} = 16$

$\frac{8e^{2x}}{8} = \frac{16}{8}$

$e^{2x} = 2$

$2x = \ln 2$

$\frac{2x}{2} = \frac{\ln 2}{2}$

$x = \frac{1}{2}\ln 2$

Example 3: Solve for x: $4^{x-1} = 64$

$4^{x-1} = 4^3$

$x - 1 = 3$

$x = 4$

Example 4: Solve for x: $e^x + 3 = 0$

$-3 \quad -3$

$e^x = -3 \rightarrow$ not defined

range: $(0, \infty)$

$x = \ln(-3)$ -3 is not in the domain of this function

A logarithmic equation contains a log or an ln. To solve, isolate the log(ln) term then write as an exponential form or raised to the base of the equation and solve for the variable. In these equations you must check your answer to be sure it makes sense. Check your answers in the original to be sure that when an answer in put into a log or ln it is not zero or a negative.

Example 5: Solve for x: $\log(3x+5)=2$

$3x+5=10^2$

$3x+5=100$

$-5 \quad -5$

$3x=95$

$\frac{3x}{3}=\frac{95}{3}$

$\boxed{x=\frac{95}{3}}$

Example 6: Solve for x: $\ln(x) = 7$

$x=e^7$

Example 7: Solve for x: $\log_3(2-x)=3$

$2-x=3^3$

$2-x=27$

$-2 \quad -2$

$-x=25$

$\frac{-x}{-1}=\frac{25}{-1}$

$\boxed{x=-25}$

Example 8: Solve for x: $\log_{10}(-x-4)=2$

$-x-4=10^2$

$-x-4=100$

$+4 \quad +4$

$-x=104$

$\frac{-x}{-1}=\frac{104}{-1}$

$\boxed{x=-104}$

Example 9: Solve for x: $\log_5(5x-1)=\log_5(x+7)$

$5x-1=x+7$
$-x \quad -x$
$4x-1=7$
$+1 \quad +1$
$\frac{4x}{4}=\frac{8}{4} \rightarrow \boxed{x=2}$

Example 10: Solve for x: $\log_2 x+\log_2(x-7)=3$

$\log_2(x(x-7))=3$
$x^2-7x=2^3$
$x^2-7x=8$
$x^2-7x-8=0$

$x^2-7x-8=0$
$(x-8)(x+1)=0$
$x-8=0 \quad x+1=0$
$\boxed{x=8} \quad \cancel{x=-1}$

Example 11: Solve for x: $\log x+\log(x-1)=\log 4x$

$\log(x(x-1))=\log 4x$
$x^2-x=4x$
$-4x \quad -4x$
$x^2-5x=0$
$x(x-5)=0$

$\cancel{x=0} \quad x-5=0$
$\boxed{x=5}$

Example 12: Solve for x: $\log_9(x-5)+\log_9(x+3)=1$

$\log_9[(x-5)(x+3)]=1$
$x^2+3x-5x-15=9^1$
$x^2-2x-15=9$
$-9 \quad -9$
$x^2-2x-24=0$
$(x-6)(x+4)=0$
$x-6=0 \quad x+4=0$
$\boxed{x=6} \quad \cancel{x=-4}$