Group: ____________ ____________ ____________ Name: Solutions

Worksheet 3. Math 231 AL1, BL1, DL1. Spring 2015. 1/28/2015.

The goal is to learn how to evaluate the trig integrals of Section 7.2.

1. Use the method introduced in the tutorial to evaluate $\int 4\sin^2 x\cos^2 x\,dx$.

This idea works on any integral $\int \sin^n x\cos^m x\,dx$ where both n and m are even.

$$\int 4\sin^2 x\cos^2 x\,dx = \int 4\left(\tfrac{1}{2}(1-\cos 2x)\right)\left(\tfrac{1}{2}(1+\cos 2x)\right)dx$$

double angle formulas

$$= \int 1-\cos^2(2x)\,dx = \int \sin^2(2x)\,dx$$

$$= \int \tfrac{1}{2}(1-\cos(4x))\,dx = \boxed{\frac{x}{2} - \frac{\sin(4x)}{8} + C}$$

2. The goal is to evaluate $\int \sin^2 x\cos^5 x\,dx$.

Technique: Rewrite as $\int \sin^2 x\cos^5 x\,dx = \int \sin^2 x\cos^4 x(\cos x\,dx)$. Use the identity $\sin^2 x+\cos^2 x = 1$ to write $\cos^4 x$ in terms of $\sin x$. Then make the substitution $u = \sin x$.

This idea works on any integral $\int \sin^n x\cos^m x\,dx$ where one of n or m is odd.

$$\int \sin^2 x\cos^5 x\,dx = \int \sin^2 x(1-\sin^2 x)^2\cos x\,dx$$

$u = \sin x$
$du = \cos x\,dx$

$$= \int u^2(1-u^2)^2\,du = \int u^2 - 2u^4 + u^6\,du$$

$$= \frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} + C = \boxed{\frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + C}$$

3. Use the idea in problem 2 to evaluate $\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$.

$$\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx = \int \frac{(1-\sin^2 x)}{\sqrt{\sin x}}\cos x\,dx = \int \frac{1-u^2}{\sqrt{u}}\,du$$

$u = \sin x$
$du = \cos x\,dx$

$$= \int u^{-1/2} - u^{3/2}\,du = 2u^{1/2} - \frac{2}{5}u^{5/2} + C$$

$$= \boxed{2\sqrt{\sin x} - \frac{2}{5}(\sin x)^{5/2} + C}$$

4. The goal is to evaluate $\int \sec x\,dx$.

This needs a trick: Multiply top and bottom of the integral by the quantity $\sec x + \tan x$. Remember that $\frac{d}{dx}\sec x = \sec x \tan x$ and that $\frac{d}{dx}\tan x = \sec^2 x$.

$$\int \sec x\,dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}\,dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$$

$u = \tan x + \sec x$

$du = (\sec^2 x + \sec x \tan x)\,dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \boxed{\ln|\sec x + \tan x| + C}$$

5. The goal is to evaluate $\int \sec^3 x\,dx$.

To do this, write $\int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx$ and integrate by parts. Don't forget that $1 + \tan^2 x = \sec^2 x$. **This idea works on any integral $\int \sec^n x\,dx$ where n is odd.**

$u = \sec x \qquad dv = \sec^2 x$

$du = \sec x \tan x\,dx \qquad v = \tan$

Let $I = \int \sec^3 x\,dx$. Then

$$I = \int \sec x \sec^2 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$(1 + \tan^2 x = \sec^2 x)$

$$= \sec x \tan x - \int \sec x(\sec^2 x - 1)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

by #4 $\rightarrow$ $= \sec x \tan x - I + \ln|\sec x + \tan x| + C$

So: $2I = \sec x \tan x + \ln|\sec x + \tan x| + C$

and $\boxed{I = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C}$