Group: _____ _____ _____ Name: _____

Worksheet 4. Math 231 AL1, BL1, DL1. Spring 2015. 1/30/2015.

1. Fill in the table.

Expression	Substitution	dx	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$dx = a\cos\theta d\theta$	$a^2 - a^2\sin^2\theta = a^2\cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$dx = a\sec^2\theta\, d\theta$	$a^2 + a^2\tan^2\theta = a^2\sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$dx = a\sec\theta\tan\theta\, d\theta$	$a^2\sec^2\theta - a^2 = a^2\tan^2\theta$

2. Evaluate the integrals using trigonometric substitution. State the necessary restriction on the angle θ. *

(a) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$

$9 - x^2 = 9\cos^2\theta \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$= \int \frac{9\sin^2\theta}{3\cos\theta}\, 3\cos\theta\, d\theta = 9\int \sin^2\theta\, d\theta$$

$$= \frac{9}{2}\int 1 - \cos 2\theta\, d\theta = \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

$$= \frac{9}{2}\arcsin\frac{x}{3} - \frac{9}{2}\sin\theta\cos\theta + C$$

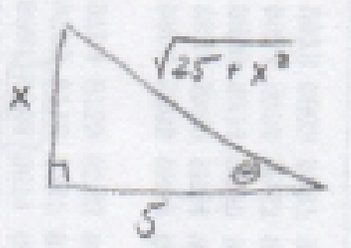
$$= \frac{9}{2}\arcsin\frac{x}{3} - \frac{9}{2}\cdot\frac{x}{3}\sqrt{\frac{9-x^2}{9}} + C = \frac{9}{2}\arcsin\frac{x}{3} - \frac{x}{2}\sqrt{9-x^2} + C$$

(b) $\displaystyle\int \frac{1}{\sqrt{25+x^2}}\,dx$

$x = 5\tan\theta \qquad dx = 5\sec^2\theta\, d\theta$

$25 + x^2 = 25\sec^2\theta$

$$= \int \frac{5\sec^2\theta\, d\theta}{5\sec\theta} = \int \sec\theta\, d\theta$$



$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{1}{5}\sqrt{25+x^2} + \frac{x}{5}\right| + C$$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$