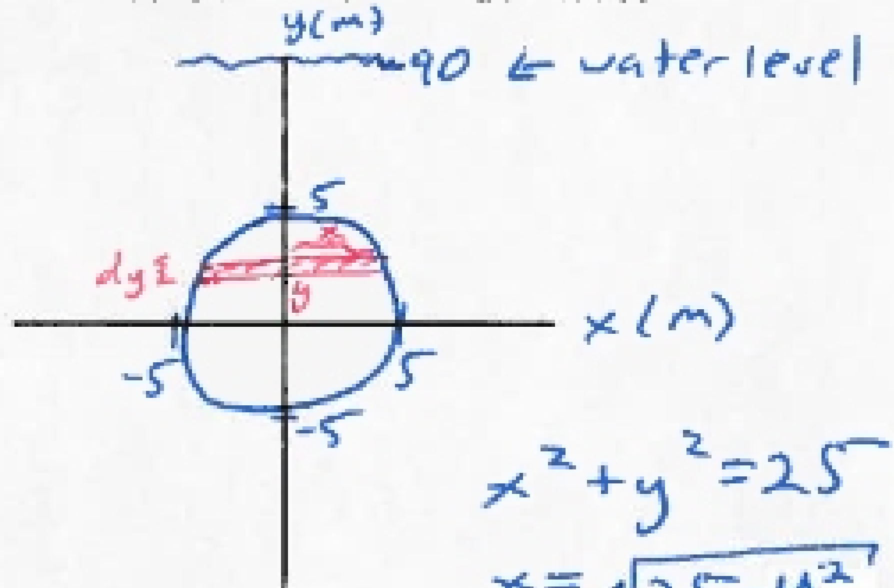
Group: ____________ ____________ ____________ Name: SOLUTIONS

Math 231 A. Fall, 2013. Worksheet 8. 10/3/13

1. The Hoover Dam near Las Vegas has "penstock gates" to control the flow of water. They are circular, approximately 5 meters in radius, and are centered 90 meters under water.

a) Make a clear diagram of this problem. Include a "ruler" to the right which clearly indicates the meaning of your coordinates.



$x^2+y^2=25$

$x=\sqrt{25-y^2}$

b) Compute the hydrostatic force on on of these gates to two significant figures. Use 9.8 m/s^2 for the gravitational constant and 1000 kg/m^3 for the density of water. **Hint:** You can evaluate all integrals which arise in your head, without any hard work.

Strip at y: area $=(2x)dy = 2\sqrt{25-y^2}\,dy$

pressure $=\rho g(90-y)$

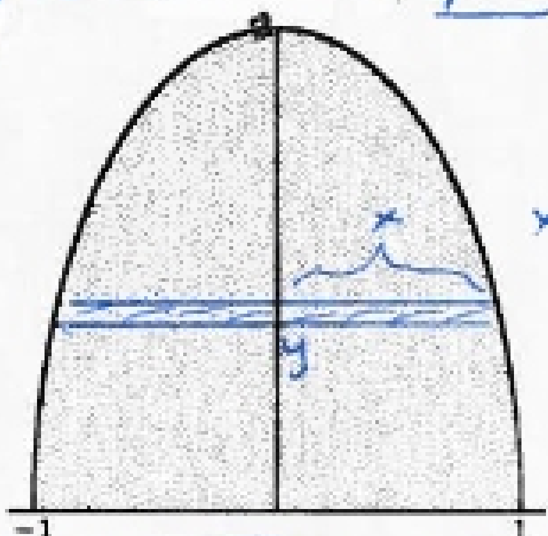
Force $=\int_{-5}^{5}\rho g(90-y)2\sqrt{25-y^2}\,dy$

$= 180\rho g\underbrace{\int_{-5}^{5}\sqrt{25-y^2}\,dy}_{\text{area of half-disk of radius 5}} - 2\rho g\underbrace{\int_{-5}^{5}y\sqrt{25-y^2}\,dy}_{\text{odd function} \Rightarrow = 0} = 180\rho g\frac{25\pi}{2} \approx \boxed{6.9\times10^7\,N}$

c) The mass of a loaded 747 airplane is approximately 400,000 kg. Find the weight of a 747 in Newtons. How many 747s would it take to provide the force you computed in part (b)?

Weight $\approx(4\times10^5)g \approx 3.9\times10^6\,N$ $\quad \frac{6.9\times10^7}{3.9\times10^6}\approx 17.7 \Rightarrow$ $\boxed{18 \text{ planes}}$

2. A lamina with area density λ kg/m^2 occupies the top half of the ellipse $4x^2+y^2=4$ as shown. You may use the fact that the area of the lamina is π m^2. Find the moments M_x and M_y about the x and y axes, respectively. Then find the coordinates $(\overline{x},\overline{y})$ of the centroid. You may use any available symmetries.



$x=\sqrt{1-(y/2)^2}$

- Strip at y: area $=(2x)dy = 2\sqrt{1-(y/2)^2}\,dy = \sqrt{4-y^2}\,dy$

  mass $=\lambda\cdot$area $=\lambda\sqrt{4-y^2}\,dy$

  $dM_x = y\cdot$mass $=\lambda y\sqrt{4-y^2}\,dy$

  $M_x=\int_0^2\lambda y\sqrt{4-y^2}\,dy = \frac{\lambda}{2}\int_0^4 u^{1/2}du = \frac{8\lambda}{3}$

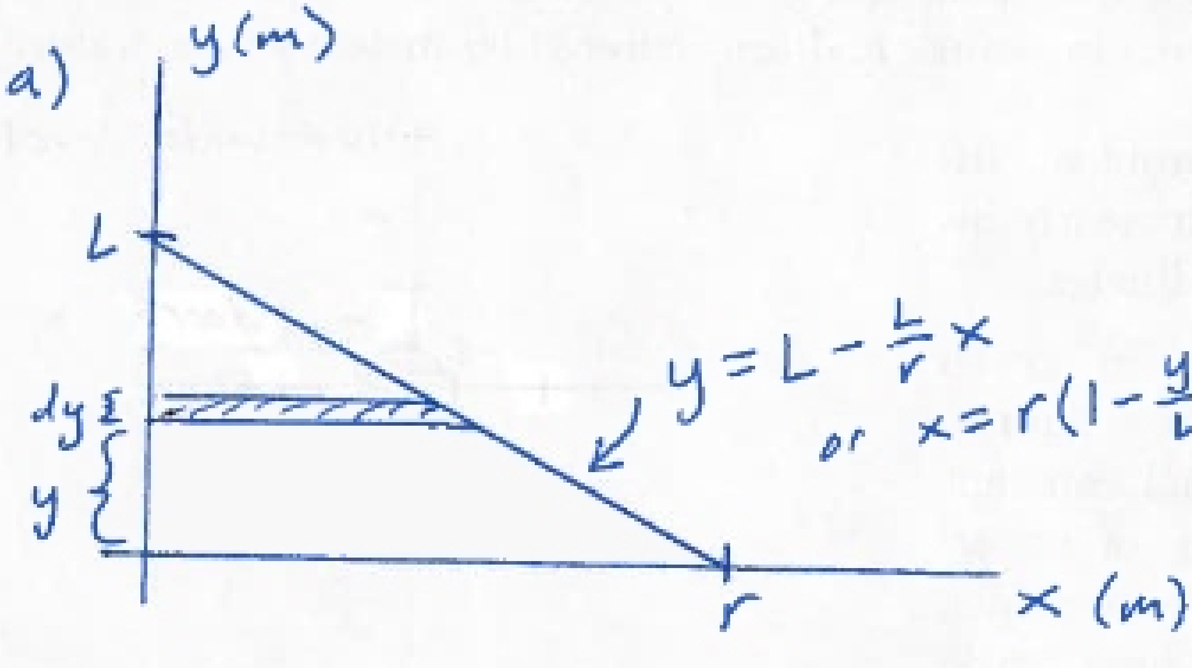
- total mass $=\lambda\cdot$area $=\lambda\pi \Rightarrow \overline{y}=\frac{M_x}{\text{mass}}=\frac{8\lambda/3}{\pi\lambda}=\frac{8}{3\pi}$
- By symmetry, $\overline{x}=0$ and $M_y=0$
- Centroid $=(0, 8/3\pi)$

3. A lamina has the shape of a right triangle of height L and base r (meters). The base lies along the x-axis. It has density ρ kg/m^2.

a) Make a careful diagram of the problem (like the one on the last page).

b) Find the moment M_x about the x axis. Your answer will involve ρ, L, and r.

a)



$y = L - \frac{L}{r}x$ or $x = r(1 - \frac{y}{L})$

b) Strip at y:

area $= x\,dy = r(1-\frac{y}{L})dy$

mass $= \rho \cdot$ area

$= \rho r (1 - \frac{y}{L}) dy$

$dM_x = y \cdot$ mass

$= y \rho r (1 - \frac{y}{L}) dy$

$$M_x = \rho r \int_0^L y(1 - y/L)\,dy = \rho r \left(\frac{y^2}{2} - \frac{y^3}{3L}\right)\Big|_0^L$$

$$= \boxed{\frac{\rho r L^2}{6}}$$