Name: Solutions

ID#: ______________________

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Problem	Weight	Score
1	35	
2	35	
3	30	
Total	100	

This test consists of three problems. Answer each problem on the exam itself; if you use additional paper, repeat the identifying information above, and staple it to the rest of your exam when you hand it in. The quality of your analysis and evaluation is as important as your answers. Your reasoning must be precise and clear; your complete English sentences should convey what you are doing.

Problem 1: (35 points)

A plant with transfer function $G_p(s)$ is compensated using a controller with transfer function $G_c(s)$ as shown in Figure 1. The experimentally determined Bode magnitude and phase plots of the plant transfer function $G_p(s)$ are shown in Figure 2.

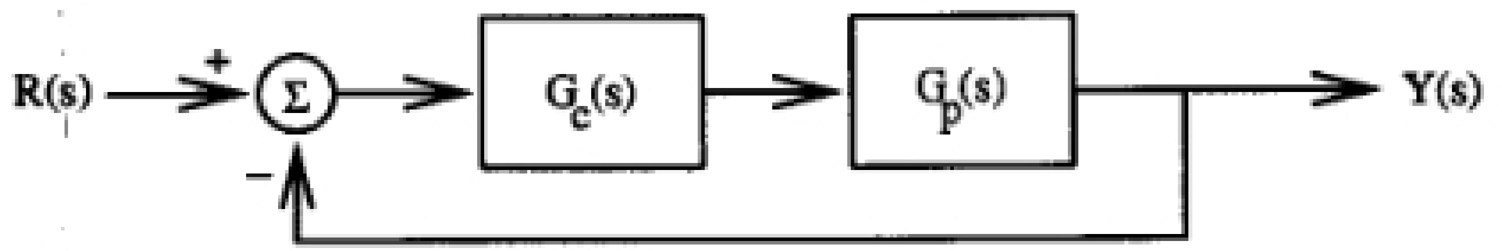

Figure 1: Feedback control system with cascade compensation.

1. (10 points) Based on the Bode plots, write a transfer function $G_p(s)$ for the plant.

$|G_p(0)| = 40\,dB$ and $\angle G_p(0) = 0^\circ \Rightarrow K_{DC} = 100$

2 peaks $\Rightarrow$ two second-order systems

$\omega_{n1} = 1.0$ rad/sec $\quad 20\log_{10}\frac{1}{2\rho_1} = 40\,dB \Rightarrow \rho_1 = .005$

$\omega_{n2} = 1000.0$ rad/sec $\quad 20\log_{10}\frac{1}{2\rho_2} = 40\,dB \Rightarrow \rho_2 = .005$

$$G_p(s) = K_{DC}\,\frac{\omega_{n1}^2}{s^2 + 2\rho_1\omega_{n1}s + \omega_{n1}^2}\;\frac{\omega_{n2}^2}{s^2 + 2\rho_2\omega_{n2}s + \omega_{n2}^2}$$

$$= 100\,\frac{1}{s^2 + .01s + 1}\;\frac{10^6}{s^2 + 10s + 10^6}$$

$$\boxed{G_p(s) = \frac{10^8}{(s^2 + .01s + 1)(s^2 + 10s + 10^6)}}$$

2. (10 points) What is the crossover frequency ω_c and the phase margin PM of the uncompensated system ?

$|G_p(j\omega_c)| = 0\,dB$ at $\boxed{\omega_c = 10 \text{ rad/sec}}$

$\angle G_p(j\omega_c) = -180^\circ \Rightarrow \boxed{PM = 0^\circ}$

3. (15 points) Design a phase lead controller

$$G_c(s) = K_o \frac{\left(1 + \frac{s}{\omega_o}\right)}{\left(1 + \frac{s}{k\ \omega_o}\right)}$$

that will yield a crossover frequency of 10 rad/sec with a phase margin greater than 60°. Sketch the open-loop transfer function of the compensated system: you may add your sketch to the Bode plots given in Figure 2.

Note that crossover frequency of $G_p(s)$ and $G_p(s)G_c(s)$ are identical (w_c = 10 rad/sec). To obtain a PM of 60°, let

$$\phi_m = 60° - \overbrace{PM_{G_p(s)}}^{\text{phase margin of uncompensated system}} + \underbrace{10°}_{\text{safety}} = 70°$$

Then

$$\boxed{K = \frac{1+\sin\phi_m}{1-\sin\phi_m} = 32.2}$$

Place $\phi_m = 60°$ of phase at w_c, that is, choose

$$\omega_m = \sqrt{K}\, w_o = w_c \quad \text{where} \quad \measuredangle G_c(j w_m) = \phi_m$$

Solving for w_o yields

$$\boxed{w_o = \frac{w_c}{\sqrt{K}} = \frac{10 \text{ rad/sec}}{\sqrt{32.2}} = 1.76 \text{ rad/sec}}$$

Finally, choose K_o so that $|G_c(jw_c) \underbrace{G_p(jw_c)}_{=1 \text{ from part 1}}| = 0\,dB$.

we need

$$|G_c(jw_c)| = 1 \Rightarrow K_o \left| \frac{1 + \frac{j\sqrt{K}w_o}{w_o}}{1 + \frac{j\sqrt{K}w_o}{Kw_o}} \right| = K_o \frac{\sqrt{1+K}}{\sqrt{1+\frac{1}{K}}} = 1$$

$$\boxed{K_o = \frac{\sqrt{1+\frac{1}{K}}}{\sqrt{1+K}} = 0.18}$$