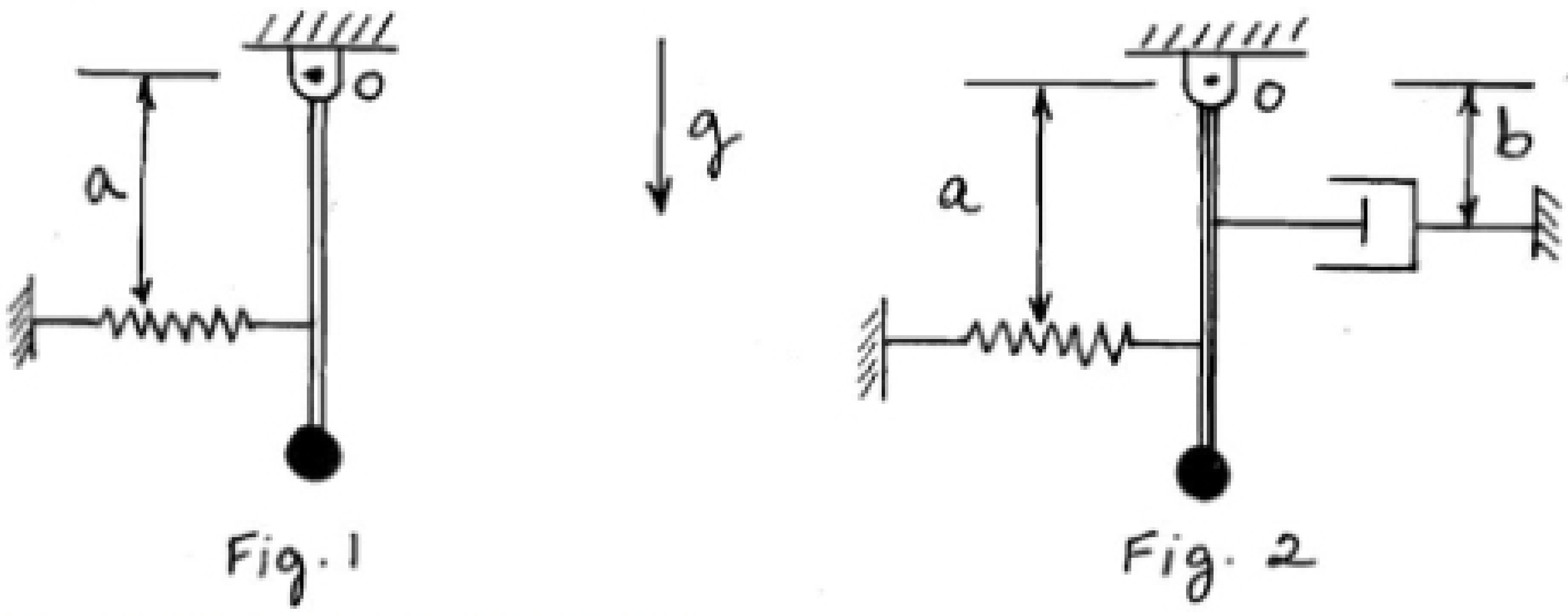
AAE 340 – Dynamics and Vibrations
Problem Set 5
Due: 2/18/13

Problem 1: The simple pendulum in Fig. 1 has mass m and a "massless" rigid rod of length L. The system can pivot about point O in a vertical plane. A spring with constant $k = \frac{mg}{L}$ is attached. Let ϕ define the orientation of the rod. The spring is unstretched when the rod is vertical and $\phi = 0^\circ$. (Assume small oscillations.)



(a) Derive the EOM for the system in Fig. 1.

$$\left\{ Ans: \ddot{\theta} + \frac{13}{9}\frac{g}{L}\theta = 0 \right\}$$

(b) Let $L = 50$ cm, $a = \frac{2L}{3}$. If $\phi(0) = 0^\circ$ and $\dot{\phi}(0) = .01\text{rad/s}$, solve for $\phi(t)$. Determine the characteristic roots, natural frequency, period, damped frequency, damping ratio. Let $\phi(0) = 0^\circ$, $\dot{\phi}(0) = .01\text{rad/s}$. Determine the total system response.

(c) Plot the system response for 3 cycles of the motion. Plot the phase diagram for 3 cycles, i.e., $\dot{\phi}$ as a function of ϕ. What does it look like?

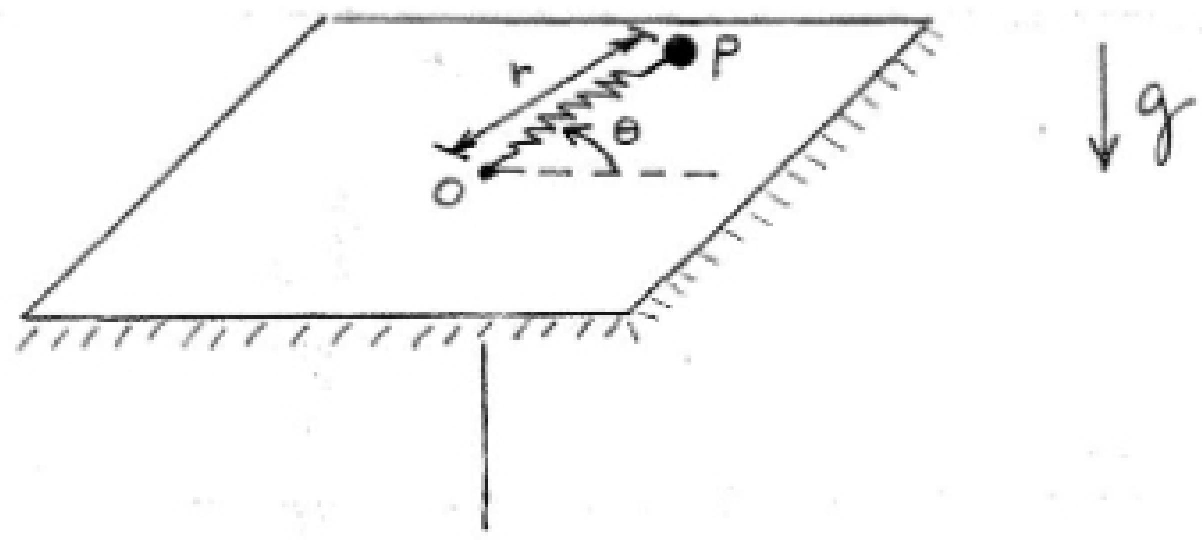
(d) Assume that a dashpot is added to the system (Fig. 2); let $c = 9m\sqrt{\frac{g}{L}}$. Let b equal two different distances: $b_1 = \frac{L}{3}$ $b_2 = \frac{2L}{3}$.

For the same initial conditions, solve for $\phi(t)$ in each case. Does the location of the dashpot change the response?

(e) For Fig. 2, plot the two responses in the same figure; add the two phase diagrams to the same plot. How do they differ for b_1, b_2?

Determine the damping ratio and the appropriate time constants. Are these quantities impacted by b? How is that reflected in the plots?

Problem 2: In the figure below, a particle P of mass m moves on a smooth, **horizontal** surface and is attached to a spring of stiffness k. The other end of the spring is fixed at point O. The spring is undeformed when $r = 0$.



(a) Derive the EOM in terms of cylindrical coordinates r and θ

$$\left[Ans: \begin{array}{c} \ddot{r} - r\dot{\theta}^2 + \frac{k}{m} r = 0 \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \end{array} \right]$$

(b) Are the differential equations linear or nonlinear? Coupled or decoupled? How do you know? Are the equations in a standard for a vibrations problem? Can you solve them analytically easily?

(c) Consider the EOM in terms of r and θ. There are 2 constants of the motion. What do they physically represent? (i.e., energy or momentum)? Write expressions for each of the constants in terms of r and θ.

(d) Let $m = 1$ kg and $k = 1$ kg/s^2. At a certain instant of time $r = 400$ mm, $\dot{\theta} = 12$ rad/s, and the radial velocity has a magnitude 1.8 m/s directed outward. At a later time $r = 600$ mm. Use the integrals/constants of the motion and find the velocity of the particle at the later time.

$$\left[Ans: \left| {}^{i}\bar{v}^{P} \right| = 5.1\,\text{m/s} \right]$$