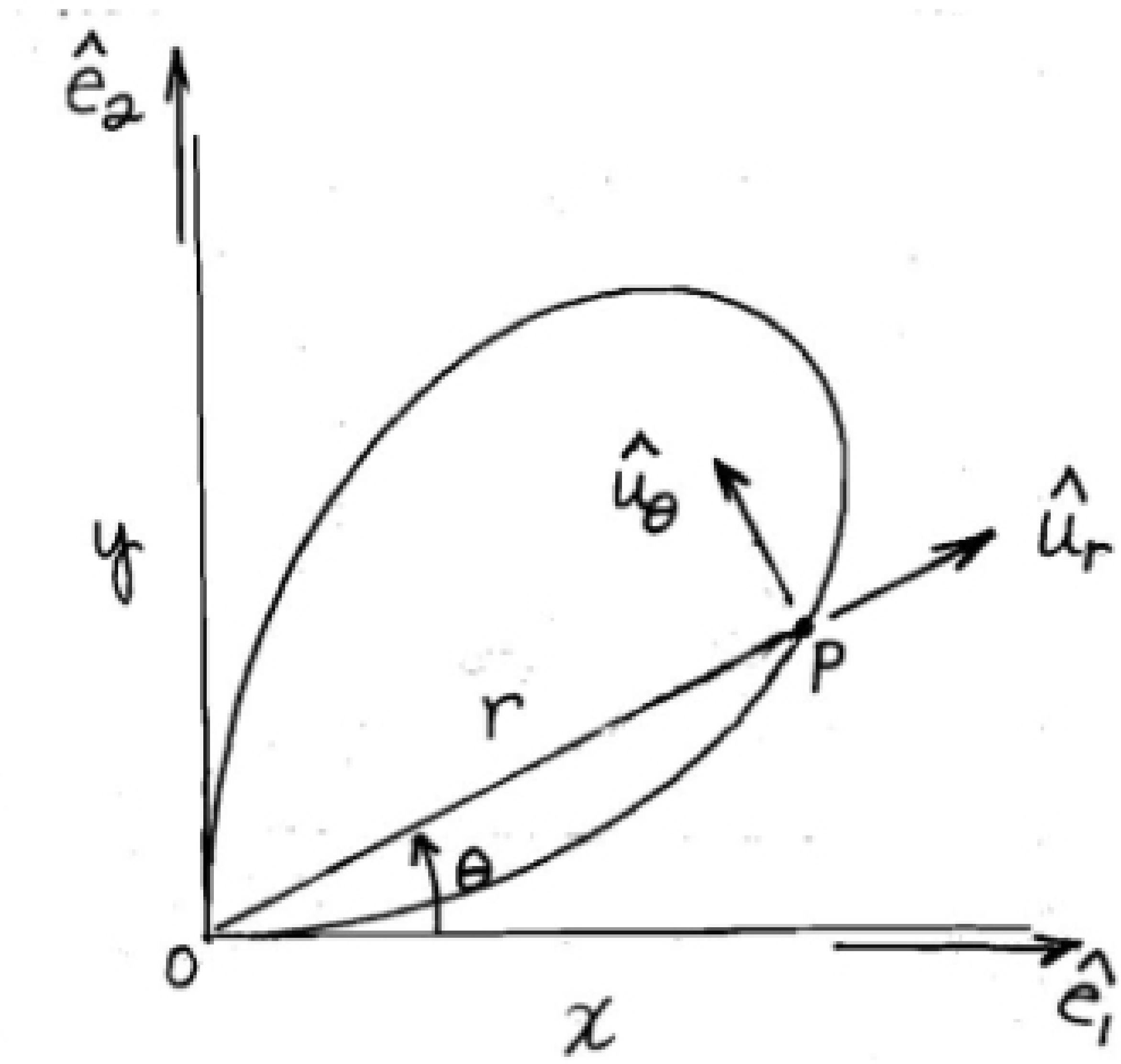
AAE 340 – Dynamics and Vibrations
Problem Set 2
Due: 1/23/13



Problem 1: A particle P departs along the curve originating from point O.

(a) Note that unit vectors $\hat{e}$ are inertial; $\hat{u}$ move and are cylindrical. Write an expression for the vector $\bar{r}^{OP}$. (Note that it can be written in terms of both r and θ.) Express $\bar{r}^{OP}$ in terms of both $\hat{e}$ and $\hat{u}$.

(b) Consider the velocity ${}^e\bar{v}^{OP}$. Derive an expression for this velocity using $\hat{e}$ as the working frame; write the final expression for ${}^e\bar{v}^{OP}$ in terms of both $\hat{e}$ and $\hat{u}$.

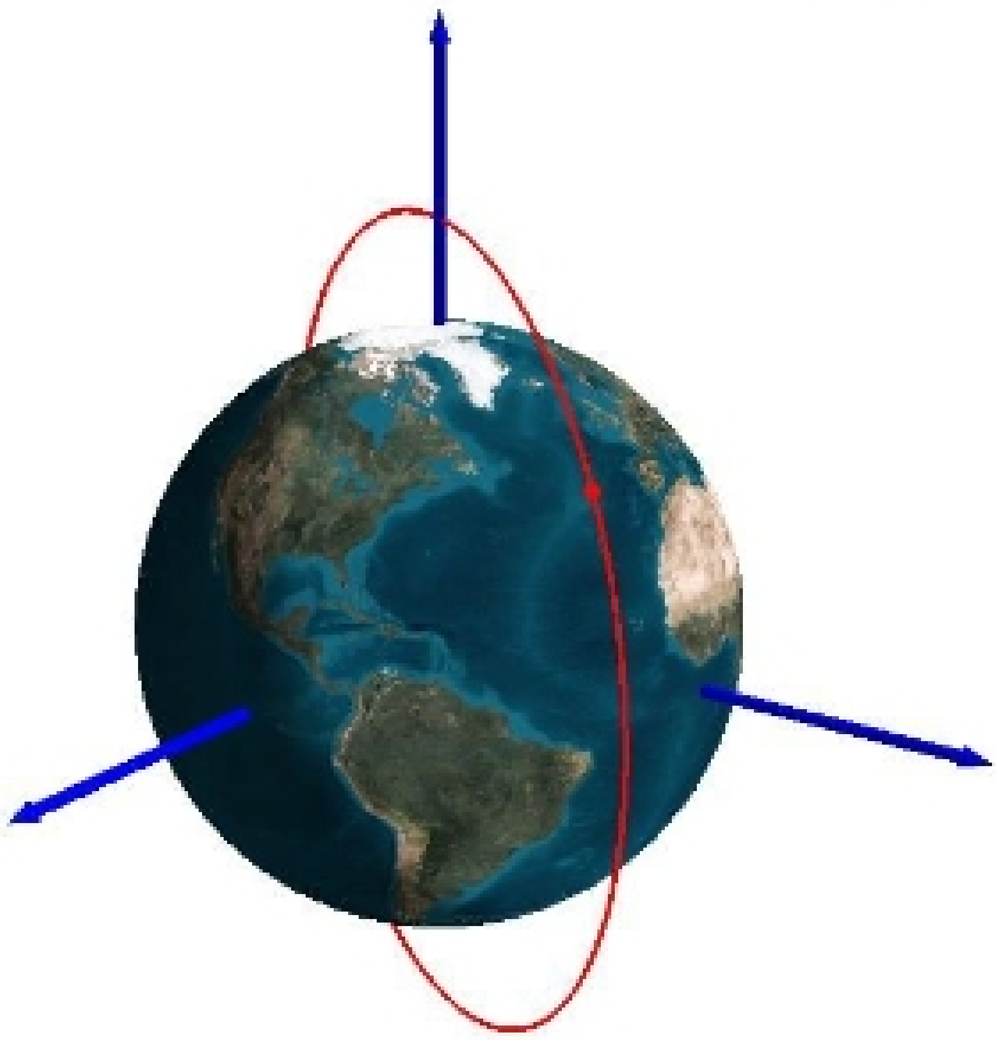
(c) Derive an expression for ${}^e\bar{v}^{OP}$ using $\hat{u}$ as the working frame; write the final expression for ${}^e\bar{v}^{OP}$ in terms of both $\hat{e}$ and $\hat{u}$.

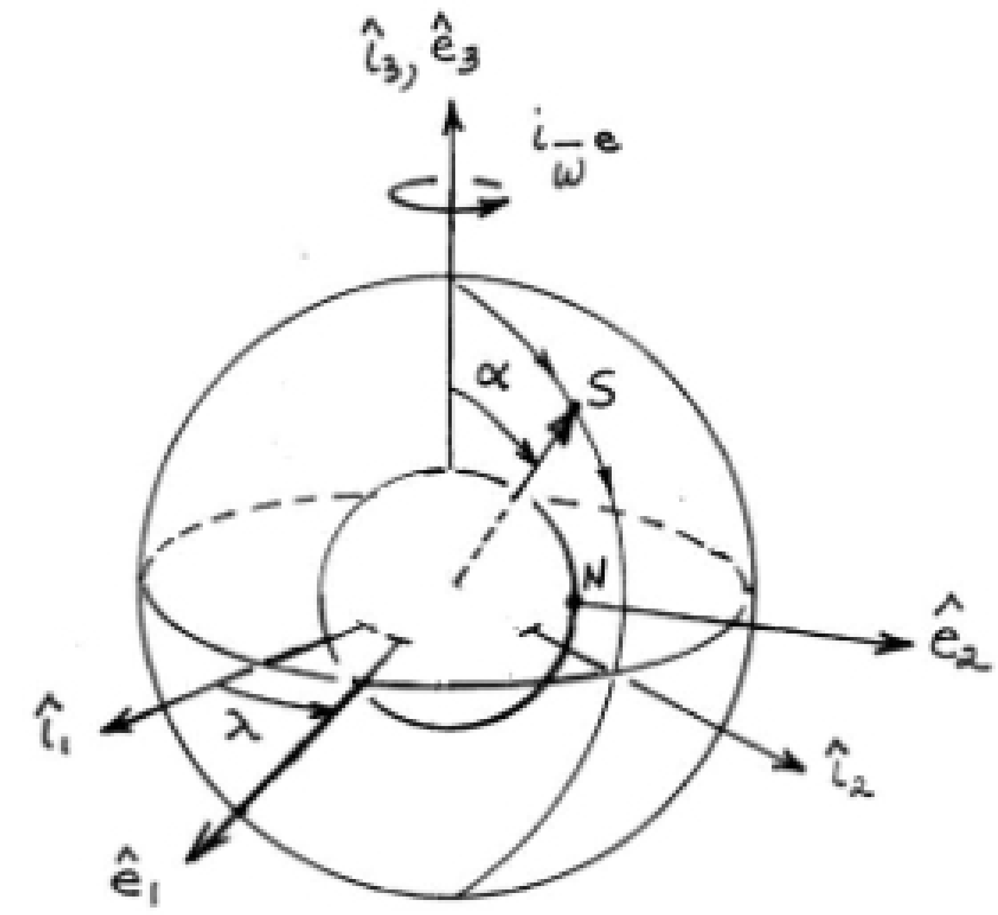
Are the expressions for ${}^e\bar{v}^{OP}$ in (b) and (c) the same? Should they be the same?

Is ${}^e\bar{v}^{OP}$ a generic velocity? How do you know?

(d) The curve in the figure is defined by the equation $r = C\sin(2\theta)$ where C is a constant and $\dot{\theta}$ is a constant value. At $t = 0$, $\theta = 0^\circ$. When time equals 6 seconds, and if $C = 4$ ft and $\dot{\theta} = 10$ deg/s, evaluate the expressions for ${}^e\bar{v}^{OP}$ (the expression in $\hat{e}$ as well as $\hat{u}$).
Compute the magnitude of ${}^e\bar{v}^{OP}$. Is the magnitude the same for both expressions, i.e., ${}^e\bar{v}^{OP}$ expressed in $\hat{e}$ as well as $\hat{u}$? Should it be the same?

Problem 2: A satellite S is moving in a circular, polar orbit about the Earth (red path on left). The constant orbital radius (from the center of the Earth) is 7000 km; the satellite velocity relative to an inertial observer ($\hat{i}_1, \hat{i}_2, \hat{i}_3$) is given in terms of a constant magnitude 7.456 km/s. Note that any orbit plane is always fixed in the inertial frame, in this case the $\hat{i}_2 - \hat{i}_3$ plane. Let $\hat{e}_1, \hat{e}_2, \hat{e}_3$ (blue axes on left) be fixed in the Earth and, thus, this set of unit vectors rotates with the Earth; the Earth rotates on its own axis once every 23.934 hours about the axis $\hat{i}_3 = \hat{e}_3$. Thus, the angle between $\hat{i}_1$ and $\hat{e}_1$ is λ and ${}^{i}\bar{\omega}^{e} = \dfrac{2\pi \text{ rad}}{23.934 \text{ hrs}}$. Note that the radius of the Earth is 6378 km.





(a) Sketch the orbit of the satellite as it appears in the $\hat{i}_2 - \hat{i}_3$ plane. Define a set of unit vectors $\hat{s}_j$ that move with the satellite. Write expressions for the unit vector relationships between $\hat{i}$ and $\hat{e}$ as well as $\hat{e}$ and $\hat{s}$. (Note that $\hat{s}$ is not truly a spherical set of unit vectors!)

(b) Assume that point O is located at the center of the Earth. Write the position vector $\bar{r}^{OS}$ and express it in terms of each of the three sets of unit vectors $\hat{e}, \hat{i}, \hat{s}$.

(c) Use the BKE and derive kinematical expressions for ${}^{i}\bar{v}^{OS}$, ${}^{i}\bar{A}^{OS}$. Express both in terms of $\hat{s}_j$. Using the given information, determine the numerical values for $\dot{\alpha}$, $\ddot{\alpha}$ and evaluate ${}^{i}\bar{v}^{OS}$, ${}^{i}\bar{A}^{OS}$. (Are these generic?)

(b) Return to the full sketch above. Recall that unit vectors $\hat{e}_1, \hat{e}_2, \hat{e}_3$ are fixed in the Earth. Use the BKE and derive general expressions for velocity and acceleration relative to an observer fixed on the Earth, that is, ${}^{e}\bar{v}^{OS}$ and ${}^{e}\bar{A}^{OS}$. Express these quantities in terms of $\hat{e}; \hat{s}$.

(c) Express ${}^{i}\bar{v}^{S}$, ${}^{i}\bar{A}^{S}$ in terms of unit vectors fixed in the Earth. Are these the same quantities as ${}^{e}\bar{v}^{S}$, ${}^{e}\bar{A}^{S}$? Should they be equal?

(d) The point N is identified as an observer fixed on the surface of the Earth at the location indicated — perhaps a tracking station. Write the position vector $\bar{r}^{NS}$.

Derive an expression for ${}^{e}\bar{v}^{NS} = \frac{{}^{e}d\bar{r}^{NS}}{dt}$. How does this expression compare to the velocities ${}^{i}\bar{v}^{OS}$, ${}^{e}\bar{v}^{OS}$? Are they the same? Does that make sense?

Evaluate this velocity ${}^{e}\bar{v}^{NS} = \frac{{}^{e}d\bar{r}^{NS}}{dt}$. What is the speed of the satellite relative to the tracking station at the instant when $\alpha = 60^{\circ}$ and $\lambda = 45^{\circ}$?