

PROBLEM 1

Given System of 2 particles (each of mass m)

(A) slides on a horizontal frictionless surface

(B) is connected to (A) by a massless, inextensible rod of length L

Find

(a) Define all the appropriate quantities.
Explain why only 2 DOF are required

(b) System FBD

Number of EOM

EOM (using sys. force eqn & moment eqn about A)

(c) Justify Energy as const of motion

Write the system linear momentum P .

Identify the constant component from the FBD

Derive expression for the two const.

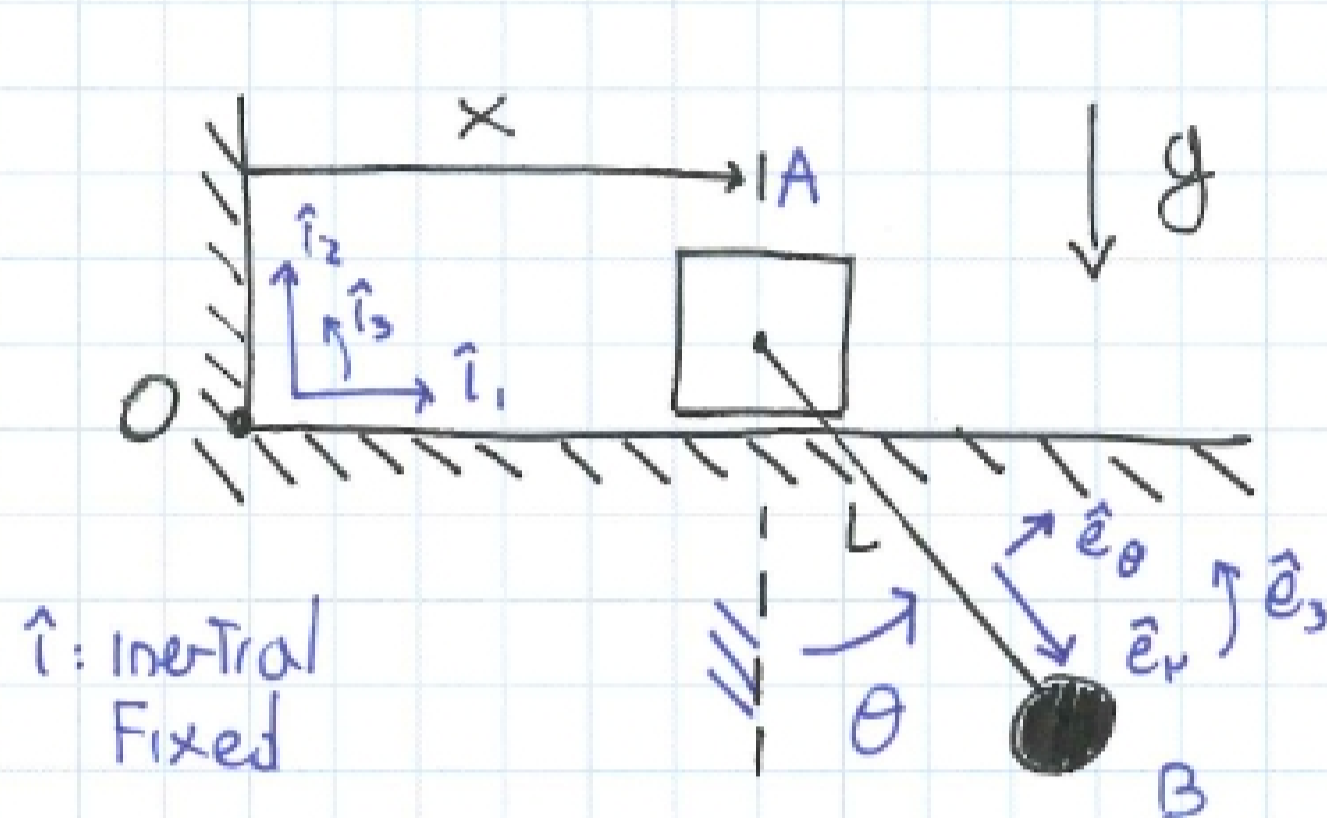
(d) Let $m = 2 \text{ kg}$, $L = 5 \text{ m}$

$$\theta(0) = 60^\circ, \dot{\theta}(0) = 0, x(0) = 1 \text{ m}, \dot{x}(0) = 0$$

Determine $\dot{\theta}, \dot{x}$ when $\theta(0) = 0^\circ$

Solution

(a) Diagram & Definitions



* VOI:

x : distance of (A) from the wall

θ : rotation of the rod w.r.t to the vertical

\hat{i} : Inertial Fixed

For a planar problem we need 2 VOI for each mass point.

We would need 4 VOI for this problem. However

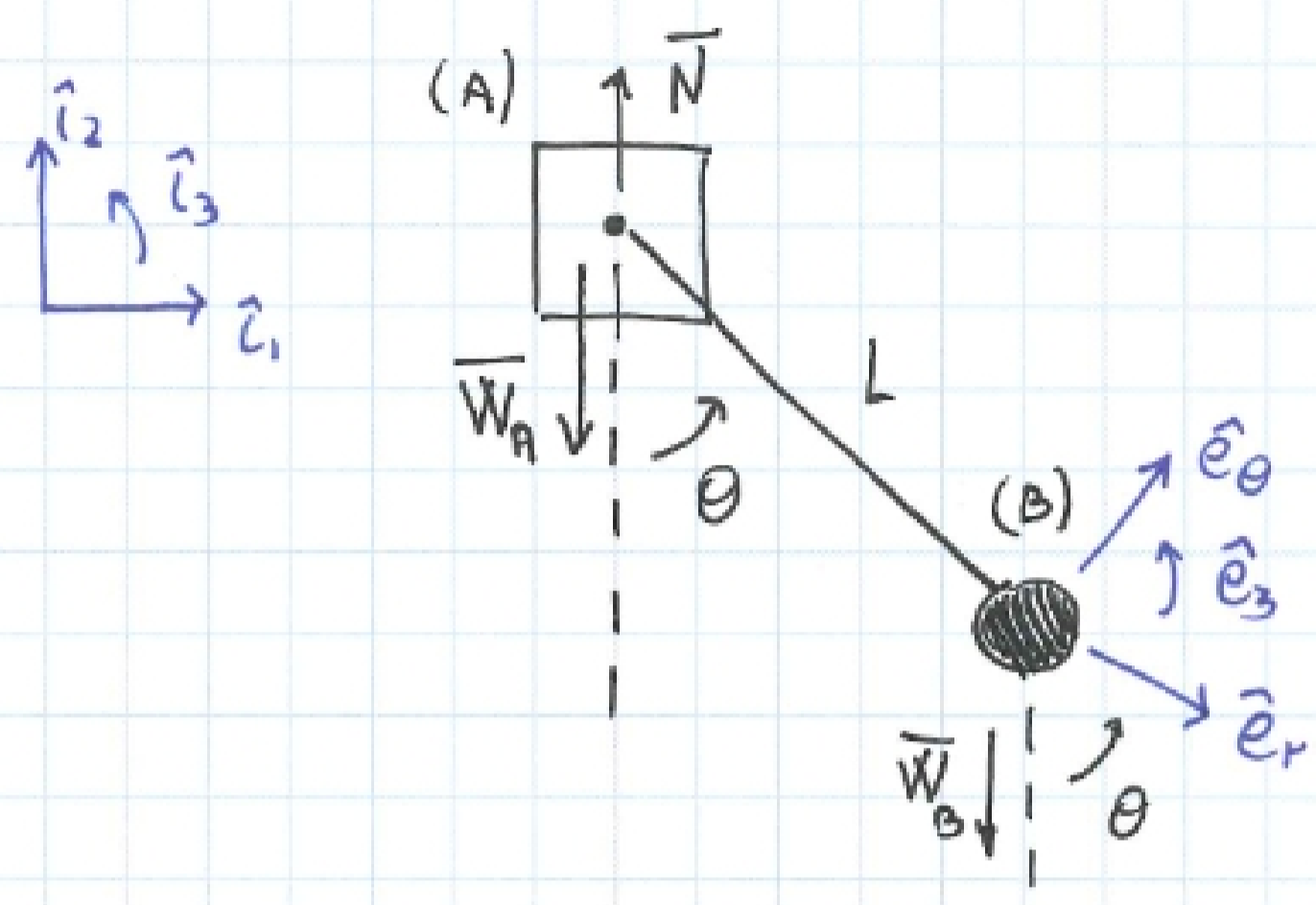
- 1) the rod constrains the relative distance b/w the two particles \rightarrow the # of VOI is reduced to 3
- 2) We ASSUME (A) remaining in contact with the floor so that (A) can only move horizontally.

(For sake of completeness (A) is free to move away from the floor, but it seems very unlikely for this problem)

With this assumption the # of VOI's reduces to 2

\rightarrow Then, 2 EOM's are required /*

(b) • FBD ^{*} For the system (internal forces do not show up)



Forces:

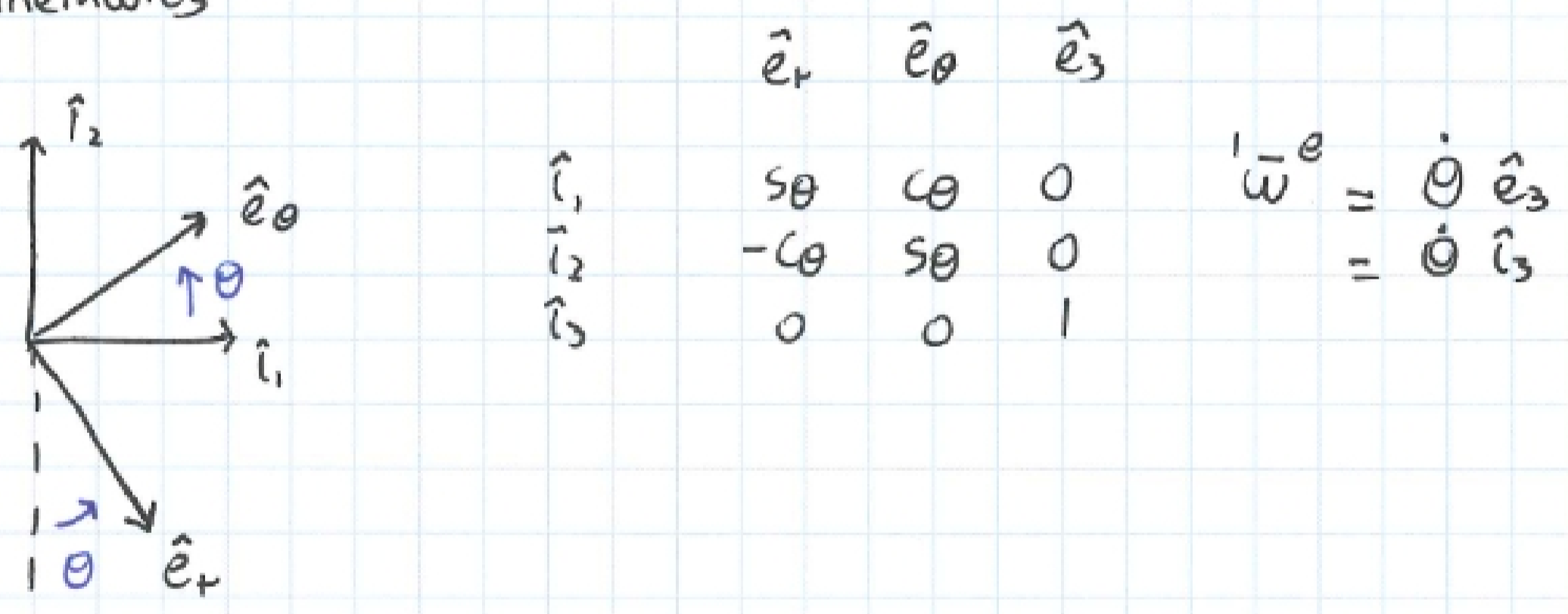
$$\bar{N} = N \hat{i}_2$$

$$\bar{W}_A = -m_A g \hat{i}_2$$

$$\bar{W}_B = -m_B g \hat{i}_2$$

$m_A = m_B = m$

• Kinematics



$$\bar{R}^{OA} = x \hat{i}_1 \quad \bar{R}^{OB} = x \hat{i}_1 + L \hat{e}_r$$

$$= x \hat{i}_1 + L \cos \theta \hat{i}_1 - L \sin \theta \hat{i}_2$$

$$\bar{R}^{OB} = (x + L \cos \theta) \hat{i}_1 - L \sin \theta \hat{i}_2$$

$$\bar{R}^{cm} = \frac{1}{2m} \left((mx + mx + mL \cos \theta) \hat{i}_1 - mL \sin \theta \hat{i}_2 \right)$$

$$= \left(x + \frac{L}{2} \cos \theta \right) \hat{i}_1 - \frac{L}{2} \sin \theta \hat{i}_2$$