

Oscillator Stability

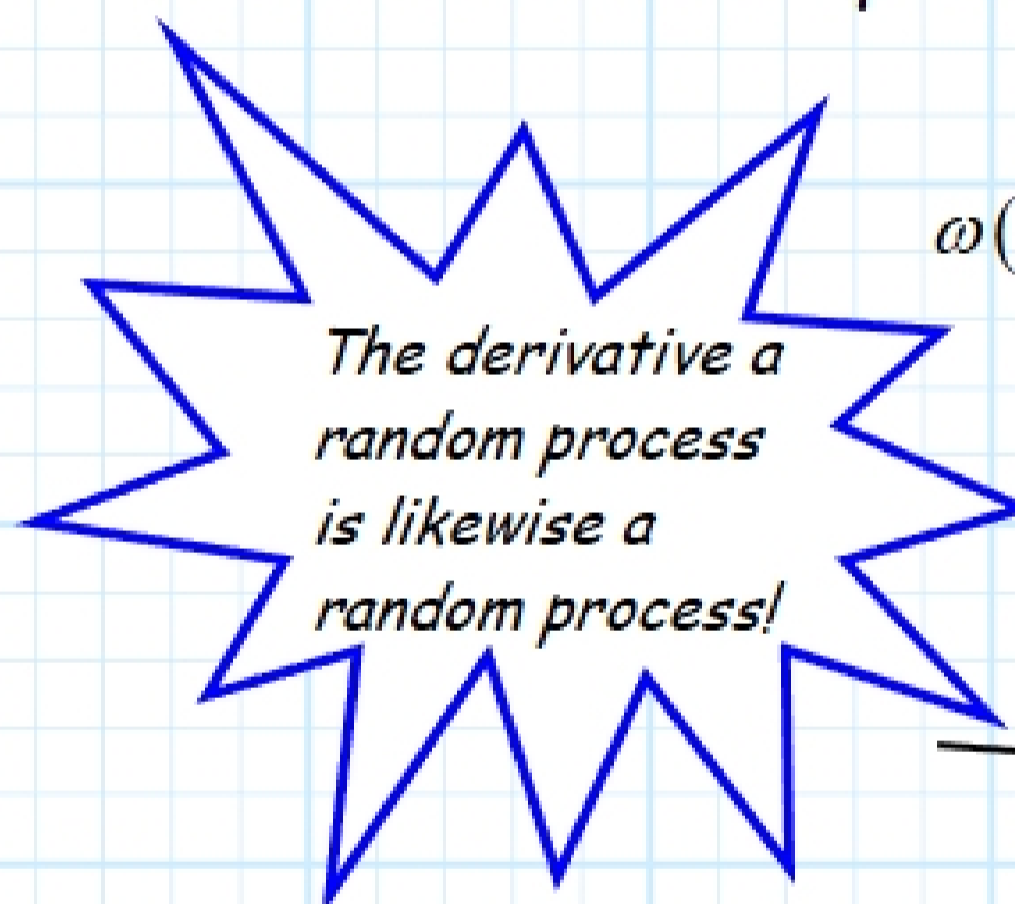
In addition to noise, spurs, and harmonics, oscillators have a problem with frequency/phase **instability**.

I.E., a better model for the oscillator signal is:

$$v_c(t) = A_c \cos[\omega_0 t + \phi_r(t)]$$

where $\phi_r(t)$ is a **random process** !

Note then the frequency will likewise be a random process:



The derivative a random process is likewise a random process!

$$\begin{aligned} \omega(t) &= \frac{d[\omega_0 t + \phi_r(t)]}{dt} \\ &= \omega_0 + \frac{d\phi_r(t)}{dt} \\ &= \omega_0 + \omega_r(t) \end{aligned}$$

An arrow points from the starburst to the $\omega_r(t)$ term in the final equation.

In other words, the frequency of an oscillator will **vary** slightly with time.

We refer to these random variations as oscillator instability, and these instabilities come in two general types:

1) Long term instabilities - These are **slow** changes in oscillator frequency over time (e.g., minutes, hours, or days), generally due to **temperature** changes and/or oscillator **aging**. For good oscillators, this instability is measured in **parts per million (ppm)**.

Parts per million is a similar to describing the instability in terms of **percentage** change in oscillator frequency. However, instead of expressing this change relative to one one-hundredth of the oscillator frequency ω_0 (i.e., one **percent** of the oscillator frequency), we express this change relative to one one-millionth of the oscillator frequency ω_0 !

A more direct way of expressing “parts per million” is “**Hz per MHz**”—in other words the amount of frequency change $\Delta\omega_r$ in **Hz**, divided by the oscillator frequency expressed in **MHz**.

For **example**, say an oscillator operates at a frequency of $f_0 = 100 \text{ MHz}$. This oscillator frequency will can (slowly) change as much as $\Delta f_r = \pm 10 \text{ kHz}$ over time. We thus say that the **long-term stability** of the oscillator is:

$$\frac{\Delta f_r (\text{Hz})}{f_0 (\text{MHz})} = \frac{\pm 10,000}{100} = \pm 100 \text{ ppm}$$

2) Short-term instabilities - The short-term instabilities of oscillators are commonly referred to as **phase noise**—a result of having **imperfect resonators**!

With phase noise, the random process $\phi_r(t)$ has very **small magnitude**, but changes very **rapidly** (e.g., milliseconds or microseconds). This is equivalent to narrow-band **frequency modulation (FM)**, and the result is a **spreading** of the oscillator signal spectrum.

Phase-noise is a very complex phenomenon, yet can be **critical** to the performance (or lack thereof) of a radio receiver. As such, it deserves its very **own** handout!