EXAMPLE 13-3A

Design of a Helical Compression Spring for Static Loading: An Alternate Approach

Problem **Design a compression spring for a static load over a known deflection with a factor of safety against yielding at shut height of at least 1.1.**

Units $ksi := 10^3 \cdot psi$

Given
Minimum force $F_{init} := 100 \cdot lbf$

Maximum force $F_{work} := 150 \cdot lbf$

Working deflection $\Delta y := 0.75 \cdot in$

Assumptions **Use the least expensive, unpeened, cold-drawn spring wire (ASTM A227) since the loads are static.**

Shear modulus $G := 11.5 \cdot 10^6 \cdot psi$

Solution **See Mathcad file EX13-03A.**

1 We will derive a design equation for this problem that will yield a value for the wire diameter that is a function of two parameters, spring index C and the ratio, α, of the clash allowance to the working deflection. To start, we write the equation for the factor of safety against yielding at shut height

$$\tau_{shut} = \frac{S_{ys}}{N_s} \qquad (a)$$

From the given data we have a desired value for the spring rate

$$k := \frac{F_{work} - F_{init}}{\Delta y} \qquad k = 66.667 \frac{lbf}{in} \qquad (b)$$

But, from equations 13.5 and 13.7, the spring rate is given as

$$k = \frac{d \cdot G}{8 \cdot C^3 \cdot N_a} \qquad (c)$$

Eliminating k from equations b and c and solving for the number of active coils, N_a, we have

$$N_a = \frac{G \cdot \Delta y \cdot d}{8 \cdot C^3 \cdot (F_{work} - F_{init})} \qquad (d)$$

Combining equations 13.5, 13.7, and 13.8b, the stress at shut height is

$$\tau_{shut} = \frac{8 \cdot k \cdot (C + 0.5)}{\pi \cdot d^2} \cdot y_{shut} \qquad (e)$$

where the shut height y_{shut} is

$$y_{shut} = y_{work} + \Delta y_{clash} = \frac{F_{work}}{k} + \alpha \cdot \Delta y \qquad (f)$$

and
$$\alpha = \frac{\Delta y_{clash}}{\Delta y}$$

Substituting equation *f* into equation *e*,

$$\tau_{shut} = \frac{8 \cdot k \cdot (C + 0.5)}{\pi \cdot d^2} \cdot \left(\frac{F_{work}}{k} + \alpha \cdot \Delta y \right) \qquad (g)$$

From equation 13.3 and Table 13-6, the torsional yield strength of the wire is

$$S_{ys} = K_m \cdot A \cdot d^b \qquad (h)$$

and K_m is the reduction factor taken from Table 13-6, expressed as as a decimal fraction.

Substituting equations *g* and *h* into *a* and solving for *d* yields our design equation

$$d = \left[\frac{8 \cdot N_s \cdot (C + 0.5) \cdot \left[F_{work} \cdot (1 + \alpha) - \alpha \cdot F_{init} \right]}{\pi \cdot K_m \cdot A} \right]^{\frac{1}{2+b}} \qquad (i)$$

Once we choose a material for the wire, the only unknowns in this equation are the parameters *C* (spring index) and α (clash allowance to working deflection ratio).

2 Assume a spring index of 8 and a clash allowance of 15% of the working deflection, then

Spring index $C := 8$

Clash allowance ratio $\alpha := 0.15$ (*j*)

3 From Tables 13-4 and 13-6 for A227 wire we have

$A := 141.04 \cdot ksi \qquad b := -0.1822 \qquad K_m := 0.60$ (*k*)

4 Using these values and equation *i* we can solve for the required wire diameter. In order to compare this solution with Example 13-3, let $N_s := 1.24$

$$d := \left[\frac{8 \cdot N_s \cdot (C + 0.5) \cdot \left[F_{work} \cdot (1 + \alpha) - \alpha \cdot F_{init}\right]}{\pi \cdot K_m \cdot A \cdot in^2}\right]^{\frac{1}{2+b}} \cdot in$$

$d = 0.192 \cdot in$

This is a preferred diameter as given in Table 13-2, so we will accept it. Notice that the term in the large square brackets has units of in^2. In order to raise this term to a fractional exponent, we must make it dimensionless by dividing by in^2 and then multiplying the result by in.

5 Calculate the mean coil diameter D from equation 13.5 for $d := 0.192 \cdot in$.

Mean coil diameter $D := C \cdot d$ $D = 1.536 \cdot in$ (l)

6 Find the direct shear factor K_s and use it to calculate the shear stress in the coil at the larger force.

Direct shear factor $K_s := 1 + \frac{0.5}{C}$ $K_s = 1.063$ (m

Stress at F_{work} $\tau_{work} := K_s \cdot \frac{8 \cdot F_{work} \cdot D}{\pi \cdot d^3}$ $\tau_{work} = 88.1 \cdot ksi$ (n)

7 Find the ultimate tensile strength of this wire material from equation 13.3 and Table 13-4 and use it to find the torsional yield strength from Table 13-6, assuming that the set has been removed and using the low end of the recommended range.

Ultimate tensile strength $S_{ut} := A \cdot \left(\frac{d}{in}\right)^b$ $S_{ut} = 190.5 \cdot ksi$ (o)

Shear yield strength $S_{ys} := K_m \cdot S_{ut}$ $S_{ys} = 114.3 \cdot ksi$ (p)

8 Find the safety factor against yielding at this working deflection from equation 13.14.

Safety factor at working deflection $N_s := \frac{S_{ys}}{\tau_{work}}$ $N_s = 1.30$ (q)

9 To achieve the desired spring rate, the number of active coils must satisfy equation 13.7, solving for N_a yields:

Number of active coils $N_a := \frac{d^4 \cdot G}{8 \cdot D^3 \cdot k}$ $N_a = 8.086$ $N_a := 8$ (r)