

The experiment: *One Red clover plant is grown in each of 25 pots (one plant per pot). Each plant (or pot) is inoculated from a culture of one of 5 different Rhizobium strains. Plants are incubated for two weeks and each Red clover plant is tested for nitrogen content.*

Ask yourself , “What is the dependent variable and what are the treatments?”

- 1) What is the experimental unit?
- 2) What is the sampling unit?
- 3) Are the treatments fixed or random?

1)  $H_0: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha$

2)  $H_1$ : some  $\alpha_i$  is different

- 3)
  - a) Assume that the observations are normally distributed about each mean, or that the residuals (i.e. deviations) are normally distributed.
  - b) Assume that the observations are independent
  - c) Assume that the variances are homogeneous

4) Set the level of type I error. Usually  $\alpha = 0.05$

5) Determine the critical value. The test is an ANOVA (F test). There are  $t=5$  treatments (the Rhizobium strains) so the numerator has  $t-1 = 4$  d.f. Each treatment has  $n=5$  observations. The error degrees of freedom will be  $t(n-1) = 5(4) = 20$  d.f.  $F_{\alpha=0.05, 4, 20 \text{ d.f.}} = 2.8661$ .

6) Obtain data and evaluate.

**The raw data for this experiment is given below.**

**The calculations needed for the ANOVA are as follows.**

The treatment sum of squares, as developed by Fisher, are converted to a "variance" and tested with an F test against the pooled error variance. In practice, the sum of squares are usually calculated and presented with the degrees of freedom in a table called an ANOVA table.

The uncorrected SS for treatments is  $USS_{Treatments} = \frac{\sum_{i=1}^t \left( \sum_{j=1}^n Y_{ij} \right)^2}{n} - n \sum_{i=1}^t \left( \frac{\sum_{j=1}^n Y_{ij}}{n} \right)^2 =$

$$\frac{144.1^2}{5} + \frac{73.2^2}{5} + \frac{119.9^2}{5} + \frac{99.6^2}{5} + \frac{66.3^2}{5} = 4152.96 + 1071.65 + 2875.20 + 1984.03 + 879.14 = 10962.98$$

The uncorrected SS for the total  $SS_{Total} = \sum_i \sum_j Y_{ij}^2 = 19.4^2 + 17.0^2 + 17.7^2 + \dots + 14.3^2 = 11235.65$

The correction factor for both terms is  $CF = \frac{\left( \sum_i \sum_j Y_{ij} \right)^2}{tn} = \frac{503.1^2}{25} = 10124.3844$

The same correction factor is used for both the SSTotal and SSTreatments, so

- The corrected SSTotal = 11235.65 - 10124.3844 = 1111.2656
- The corrected SSTreatment = 10962.98 - 10124.3844 = 838.5976

The error term (SSError) is calculated by either;

- subtracting the Uncorrected SSTotal from the Uncorrected SSTreatment.
- subtracting the Corrected SSTotal from the Corrected SSTreatment.

c) the pooled within group variance  $SSE_{Error} = S_p^2 = \frac{SS_1 + SS_2 + SS_3 + SS_4 + SS_5}{V_1 + V_2 + V_3 + V_4 + V_5}$

$$= 1111.2656 - 838.5976 = 11235.65 - 10962.98 = 272.668$$

**The Analysis of Variance done in EXCEL is given below.**

ANOVA: Single Factor
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Our ANOVA analyses will be done with PROC MIXED and PROC GLM. There is a PROC ANOVA, but it is a subset of PROC GLM.

### Homogeneity of Variance

Your textbook discusses one test by Hartley. It is one of the simplest tests, but not usually the best. To do this test we calculate the largest observed variance divided by the smallest observed variance. This statistics is tested with a special table by Hartley (Appendix Table 5.A in your Freund & Wilson textbook). For our experiment above the variances were 33.642, 16.943, 14.267, 1.277 and 2.038. Calculate ratio of the largest individual treatment variance to the smallest variance.  $F_{\max} = S_{\max}^2 / S_{\min}^2 = 33.642 / 1.277 = 26.34455756$ . The critical values are 3.52 for  $\alpha=0.05$  and 4.6 for  $\alpha=0.01$ . The variances do not appear homogeneous in this case.

A number of other tests are available in SAS, but only for a simple CRD (i. e. a One-way ANOVA). These test are briefly discussed below.

To get all of the tests available in SAS, use the following statement following PROC GLM.