

22S:138  
Bayesian Statistics

Calibration Experiments and Review  
of Probability

Lecture 2  
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Calibration experiments

- *calibration experiment* — a scale used to assess a person's degree of belief that a particular event will occur (or has occurred)
- All outcomes of the calibration experiment must be *equally likely* in the opinion of the person whose subjective probability is being assessed
  - Example: imagine that I promise to pay you \$100 if the roll of a 6-sided die comes up the number you call
  - If you are indifferent as to which number you call, the 6 possible outcomes are equally likely for you

Assessing subjective probability about events

- We may sometimes need to quantify our subjective probability of an event in order to make a decision or take an action.
- Example:
  - You have been offered a job as a statistician with a marketing firm in Cincinnati.
  - In order to decide whether to accept the job and move to Cincinnati, you wish to quantify your subjective probability of the event that you would like the job and would like Cincinnati
  - (We will talk about Bayesian decision theory later in the semester.)

- Calibration experiments may be useful if
  - person is not knowledgeable or comfortable with probability
  - person is uncertain as to his/her opinion about the event
- principle of using a calibration experiment to assess subjective probability
  - Person is offered a choice of 2 ways of winning a prize:
    - \* through a realization of the calibration experiment with known probability of success
    - \* through the occurrence of the event of interest.
  - The calibration experiment is adjusted at successive steps

**Example:**

Using a “chips-in-a-bowl” experiment to assess your subjective probability regarding event A: that the Department of Physics at Florida State University has more than 2 female faculty members.

- Experiment is having a blindfolded person draw one chip at random from a bowl containing chips of the same size and shape
- Let  $P_s(A)$  denote your subjective probability that event A has occurred.

- The bowl contains 3 green chips and 1 red chip. You may choose Game 1 or Game 2.

If you choose Game 2, then I conclude that your

$$0.25 < P_s(A) < 0.50$$

- Then I may go on to Step 3:  
Now the bowl contains 5 green chips and 3 red chips. You may choose Game 1 or Game 2.

If you choose Game 1, I conclude that

$$0.25 < P(A) < 0.375$$

- Step 1: The bowl contains 1 green chip and 1 red chip.

Imagine that you may choose 1 of two games.

1. I will be blindfolded and draw one chip at random. I will pay you \$100 if the chip drawn is red. I will pay you nothing if it is green.
2. I will pay you \$100 if the Physics Dept at FSU has more than 2 female faculty. I will pay you nothing if it has 2 or fewer.

If you choose Game 1, then I conclude that

$$0 < P_s(A) < 0.5$$

So I construct Step 2 as follows.

- We continue until it becomes too difficult for you to choose between the two games.
- How the chips are set up in the bowl at each step is determined by your answer at the preceding step.
- Comments:
  - The payoffs have to be imaginary, because we wish to use this procedure for assessing unverifiable probabilities.
  - Luckily, a high degree of accuracy in assessing subjective probabilities usually is not needed.

## Quick review of probability

- **event**: any outcome or set of outcomes of a random phenomenon
  - the basic element to which probability can be applied
  - usual notation is a capital letter near beginning of alphabet
  - example: random phenomenon is that we are drawing a patient at random from a huge database of patients insured by an HMO
    - \* event  $A$  is the event that the patient we draw is under 6 years of age
  - we will denote the probability of event  $A$  as  $P(A)$
- **sample space  $S$** : the set of all possible outcomes of a random phenomenon
  - $P(S) = 1$

- **complement** of an event  $A$  is the event “not  $A$ ”
  - notated  $A^C$  or  $\bar{A}$
  - $A \cup \bar{A} = S$
- the **null event** – an event that can never happen
  - notated  $\emptyset$
- events  $A$  and  $B$  are **disjoint** or **mutually exclusive** if they cannot occur together
  - i.e. if  $A \cap B = \emptyset$
  - example: event  $A$  is that the patient we draw is under 6 years of age, and event  $B$  is event that the patient is 6 to 11 years of age

- **intersection** of two events  $A$  and  $B$  is the event “both  $A$  and  $B$ ”
  - notated  $A \cap B$
  - example: if event  $B$  is the event that the patient we draw weighs at least 150 pounds, then  $A \cap B$  is the event that the patient we draw is under 6 years of age and weighs at least 150 pounds
- **union** of two events is event “either  $A$  or  $B$  or both”
  - notated  $A \cup B$
- A set of events  $A_1, A_2, A_3, \dots$  are **exhaustive** if

$$A_1 \cup A_2 \cup A_3 \cup \dots = S$$

- additive rule of probability
    - if two events  $A$  and  $B$  are mutually exclusive, then
- $$P(A \cup B) = P(A) + P(B)$$
- $P(A^C) = 1 - P(A)$