Induction Step

Let $l_{ij}^{(k)}$ denote the length of the shortest path with at most k edges from i to j. Then

$$l_{ij}^{(k+1)} = \min_{1 \le h \le n}(l_{ih}^{(k)} + l_{hj}^{(1)})$$

All-Pairs Shortest Paths

Given a digraph $G=(V,E)$, find shortest path from s to t for all pairs $\{s,t\}$ of nodes.

Theorem

In $L(G)^{n-1}$, each element $l_{ij}^{(n-1)}$ is the length of the shortest path from i to j.

Proof Each Shortest path contains at most $n-1$ edges.

How to Compute $L(A)^{(n-1)}$?

$n \leftarrow |V|;$

$m \leftarrow 1;$

$L^{(1)} \leftarrow L(G);$

while $n-1 > m$

  do $L^{(2m)} \leftarrow L^{(m)} \circ L^{(m)}$ and

    $m \leftarrow 2m;$

return $L^{(m)}$

Time $O(n^3 \lg n)$