EE599-020
Audio Signals and Systems

Wave Basics

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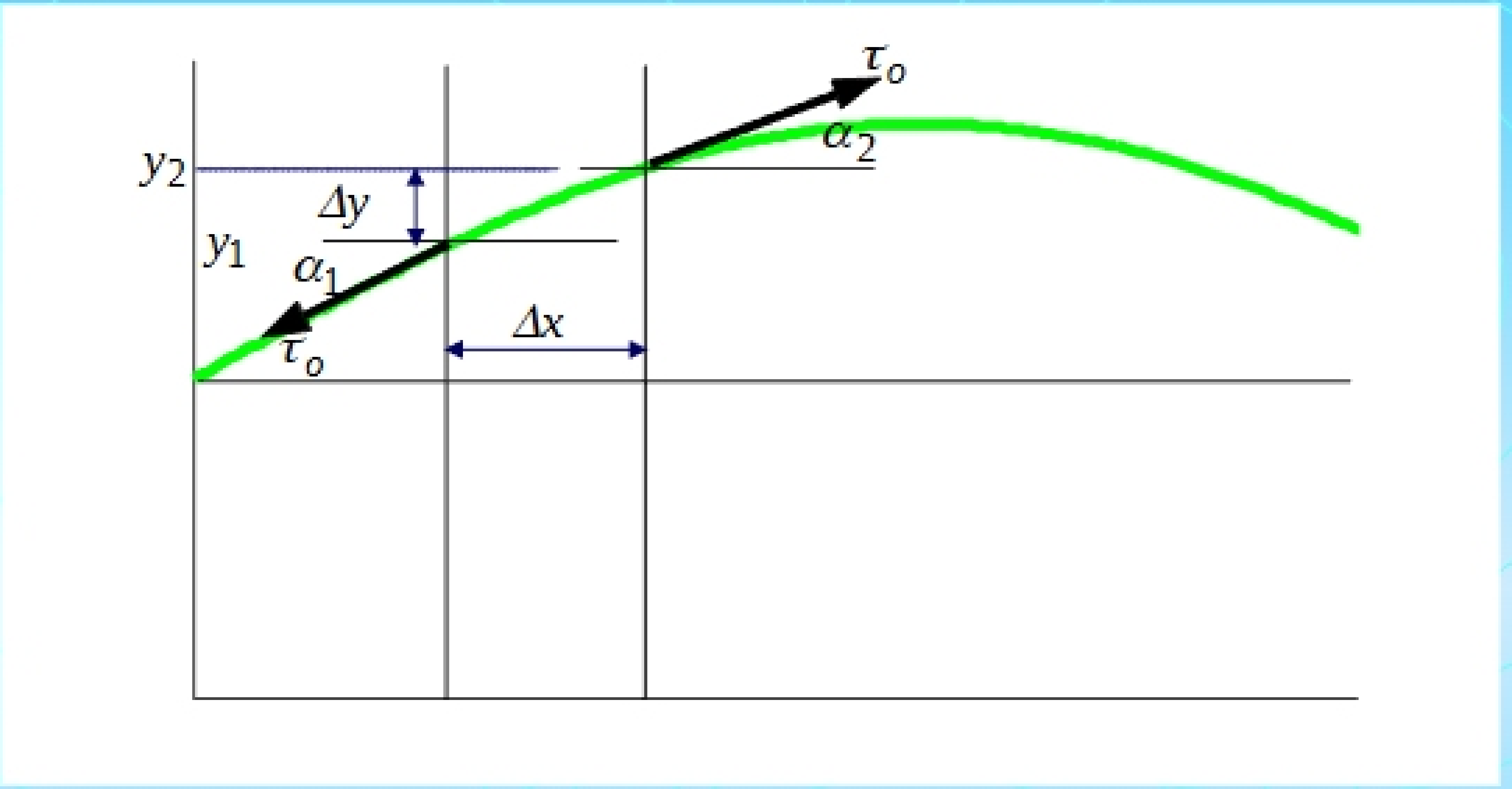
Vibrating String



Given an elastic string is displaced in the y-direction, let λ_o be the mass per unit length, assume tension τ_o is constant and independent of position, angles (α_1 and α_2) are small, motion is limited to *x-y* plane, and element Δx of the string is only displaced in the y-direction. Derive the equation governing the motion of the string.

Vibrating String

Differential forces in x and y direction from tension τ_o are given by $F_x = \tau_o(\cos(\alpha_2) - \cos(\alpha_1))$ $F_y = \tau_o(\sin(\alpha_2) - \sin(\alpha_1))$

For small α, F_x is negligible and $\sin(\alpha) = \alpha = dy/dx$.

Therefore Newton's second law for y-direction results in:

$$\lambda_o \Delta x \frac{\partial^2 y}{\partial t^2} = \tau_o \left[\frac{\partial y_2}{\partial x} - \frac{\partial y_1}{\partial x} \right]$$

$$\lambda_o \frac{\partial^2 y}{\partial t^2} = \tau_o \lim_{x \to 0} \frac{1}{\Delta x} \left[\frac{\partial y_2}{\partial x} - \frac{\partial y_1}{\partial x} \right]$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\tau_o}{\lambda_o} \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial x^2}$$