Rajan's Book Chapter 3: Structural Design Fundamentals

What is Design?

Design – a process by which an acceptable solution is obtained.

-- Feasible solution is acceptable, but desirable to have a minimum cost design. Optimization techniques can be used to achieve this objective.
-- Mixture of art and science.

Material Behavior

Stress and strain
Shear stress and strain

Material Properties

Linearly elastic range
Initial yielding – perfectly plastic range
Strain hardening phase
Ultimate stress
Fracture

Stress-Strain Relationship

Young's modulus (modulus of elasticity), E
Shear modulus, G
Poisson's ration, ν

Principal Stress and Strain

Stresses on a plane at an angle θ
Maximum direct stress
Maximum shear stress
Mohr's circle to represent state of stress at a point

Stress and Strain Computations

Need cross-sectional properties: centroid, area, moment of inertia
Axial force: N
Axial stress = force /area = N_x/A
Bending moment: M_z

$$\sigma_x = -\frac{M_z y}{I_z}$$

Bending stress: $(\sigma_x)_{max} = \frac{M_z}{S}$

$$Section\ modulus, S = \frac{I_z}{y_{max}}$$

For positive values of y, the moment gives compressive stress.

Shear force: Q

$$\tau_{xy} = \frac{V_y Q}{I_z t}$$

Shear stress: $(\tau_{xy})_{max} = \frac{V_y}{SF}$

$$Shear\ factor, SF = \frac{I_z}{(t/Q)_{min}}$$

Combined stress: $\sigma_x = \frac{N_x}{A} \mp \frac{M_z y}{I_z}$

Theories of Failure

Some causes of structural failure:
yielding
low stiffness
buckling
crushing
fracture.

Failure criteria:

Von Mises Criterion: Octahedral shearing, strain energy density of distortion

$$\tau_{oct} \geq \frac{\sqrt{2}}{3}\bar{\sigma}$$

Maximum Principal Stress Criterion:

$$\sigma_1 \geq \bar{\sigma}$$

Buckling:

$$\sigma_{cr} = \frac{\pi^2 EI}{A(KL)^2} = \frac{\pi^2 E}{(KL/r)^2}$$

$r = radis\ of\ gyration,\ r^2 = I/A$

$\lambda = L/r,\ slenderness\ ratio$