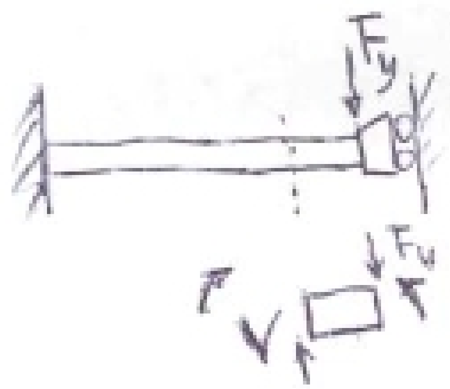
Structures

- Agenda:
  - Example: cantilever beams
  - Stress issues
    - Cantilevers
    - Doubly-supported beams
    - Buckling

- Reading: Senturia, Ch. 9, pp. 216-238.

Example Solutions

- Guided-end beam with concentrated load



$$V = +F_y$$

$$\frac{d^3 w}{dx^3} = -\frac{V}{EI} = -\frac{F_y}{EI}$$

- Integration,

$$\frac{d^2 w}{dx^2} = -\frac{F_y}{EI} x + A$$

$$\frac{dw}{dx} = -\frac{F_y}{2EI} x^2 + Ax + B$$

$$w(x) = -\frac{1}{6}\frac{F_y}{EI} x^3 + \frac{1}{2} A x^2 + Bx + C$$

Boundary conditions:

$$w\big|_{x=0} = 0 \rightarrow C = 0$$

$$\frac{dw}{dx}\Big|_{x=0} = 0 \rightarrow B = 0$$

$$\frac{dw}{dx}\Big|_{x=L} = 0 \rightarrow A = +\frac{F_y}{2EI} L$$

$$\Rightarrow w(x) = -\frac{1}{6}\frac{F_y}{EI} x^3 + \frac{1}{4}\frac{F_y}{EI} L x^2$$

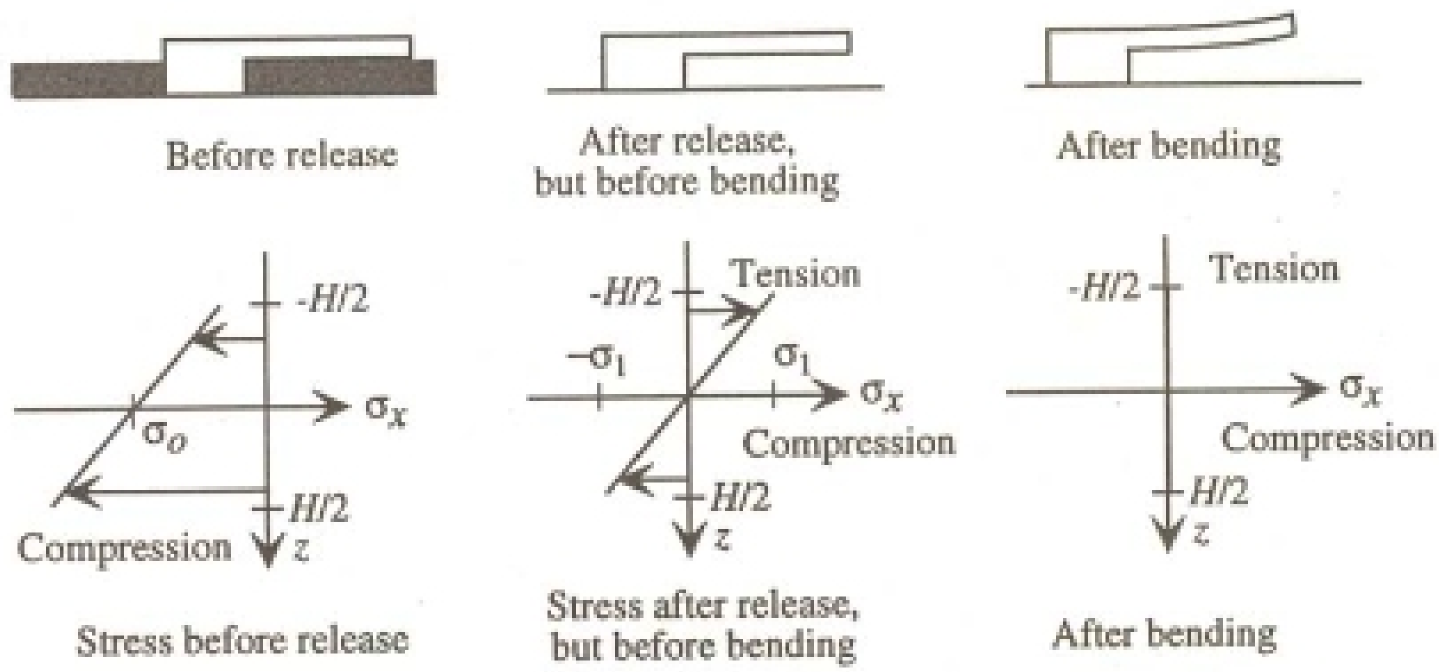
$$= \frac{1}{4}\frac{F_y L}{EI} x^2 \left(1 - \frac{x/L}{3/2}\right)$$

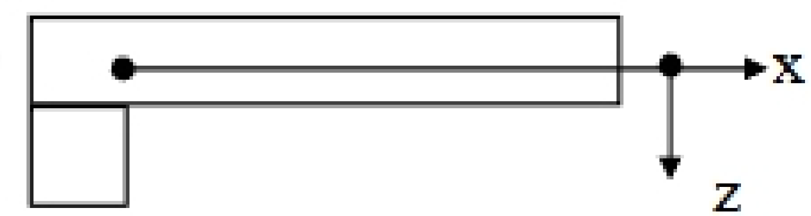
$$\Rightarrow w_{max} = \frac{1}{12}\frac{F_y L^3}{EI} \qquad I = \frac{1}{12} W H^3$$

$$= \frac{F_y L^3}{12 E \cdot \frac{1}{12} W H^3} = F_y \cdot \left(\frac{L^3}{EWH^3}\right)$$

$$\Rightarrow k_y = \frac{F_y}{w_{max}} = \frac{EWH^3}{L^3}$$

Stress Gradient in Cantilevers





- Compressive stress
- Stress gradient

$$\sigma(z) = \sigma_0 - \frac{\sigma_1}{H/2} z$$

$$M(x) = \int_{-H/2}^{H/2} \sigma(z) \cdot (W \cdot dz) \cdot z$$

$$= -\frac{1}{6} \sigma_1 W H^2$$

$$M(x) = -\frac{EI}{\rho(x)}$$

$$\rho(x) = \frac{1}{2} \frac{EH}{\sigma_1}$$

$$\rho(x) = \frac{1}{2} \left(\frac{E}{1-\upsilon} \right) \frac{H}{\sigma_1}$$