X3.17a **2050**

PHYS-205(17) (Kaldon-20639) Name **S O L U T I O N**

WMU-Summer I 2006

Exam 3A - 100,000 points + 20,000 ☆ points *Don't Forget Your Paper!*

[illegible]

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions ☐ #2a ☐ #2b ☐ #2c ☐ #2c

EXAM 3 [FORM - A]

PHYS-2050 (KALDON-17)

SUMMER I 2006

WMU

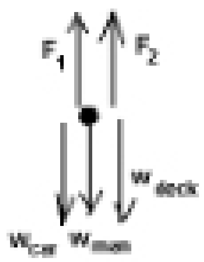
Shouldn't the game Twister be called "Torquer"?

Roving the Red Planet (36,000 points)

1.) (a) Three summers ago NASA sent the first of two rover probes to Mars – *Spirit* and *Opportunity* are still at work! To get to Mars, a 400. kg probe first has to leave the Earth. On Earth the probe weighs 3920 N. What would this 400. kg probe weigh on Mars (mass = 6.42 × 10^{23} kg ; radius = 3.37 × 10^{6} m)? $G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$

$$F_G = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11}\ {}^{N\,m^2}/_{kg^2}\right)\left(6.42\times10^{23}\ kg\right)\left(400.\ kg\right)}{\left(3.37\times10^{6}\ m\right)^2} = 1508 N$$

To launch a rocket to Mars, an intrepid NASA scientist/engineer first has to get to work in his car. The parking deck has a mass of 11,400 kg and is 20.0 m wide. Support Piers 1 and 2 are located 3.00 m from the ends. The 1950 kg car is 6.00 m from the left and our 1320 N scientist/engineer is 3.00 meters from the center-of-mass of the car as shown. Find the support forces (b) F_1 and (c) F_2. *To get full credit, you must include the F.B.D. and the F.R.D.*



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F1 is 0 meters from pivot.
Car is 3.00 meters from pivot.
Man is 6.00 meters from pivot.
Deck c.m. is 7.00 meters from pivot.
F2 is 14.0 meters from pivot.
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$$w_{car} = m_{car} g = (1950kg)(9.81m/s^2) = 19{,}130N$$
$$w_{man} = 1320N$$
$$w_{deck} = m_{deck} g = (11{,}400kg)(9.81m/s^2) = 11{,}800N$$
$$\sum F_y = F_1 + F_2 - w_{car} - w_{man} - w_{deck} = 0$$
$$\sum \tau = F_2(14.0m) - w_{car}(3.00m) - w_{man}(6.00m) - w_{deck}(7.00m) = 0$$
$$F_2(14.0m) = w_{car}(3.00m) + w_{man}(6.00m) + w_{deck}(7.00m)$$
$$F_2 = \frac{(19{,}130N)(3.00m) + (1320N)(6.00m) + (11{,}800N)(7.00m)}{14.0m}$$
$$= 10{,}570N$$
$$F_1 = w_{car} + w_{man} + w_{deck} - F_2$$
$$= 19{,}130N + 1320N + 11{,}800N - 10{,}570N$$
$$= 21{,}680N$$

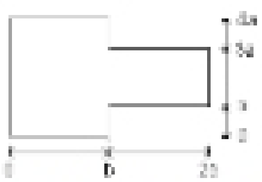
(d) Our scientist/engineer thinks he's got a powerful car – it's got 245 hp (182,800 W). If he wants to get his car from rest up to 60.0 mph (26.8 m/s), how much time would this take, assuming he can use all that power?

$$W = \Delta K = K_f - K_i = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2}(1950kg)(26.8m/s)^2 = 700{,}300J$$
$$P = \frac{W}{t}$$
$$t = \frac{W}{P} = \frac{700{,}300J}{182{,}800W} = 3.831sec$$

(e) When our scientist/engineer gets to his destination, he pulls on the parking brake with a force of 127 N. The parking brake has a 1.00 m length of cable ($Y = 10.0 \times 10^{10}$ N/m², cross-section $A = 9.00 \times 10^{-6}$ m²). How much does the cable stretch?

$$Y = \frac{F/A}{\Delta L/L_0} = \frac{F L_0}{A \Delta L}$$
$$\Delta L = \frac{F L_0}{A Y} = \frac{(127N)(1.00m)}{(9.00 \times 10^{-6} m^2)(10.0 \times 10^{10} N/m^2)}$$
$$= 0.0001411m = 1.411 \times 10^{-4} m$$

A Notch Above the Usual (30,000 points)



2.) (a) A plate of mass m has sides of $4a$ and $2b$. Find the center of mass coordinate x_{cm} by integrating $x_{cm} = \frac{1}{M}\int x\,dm$, using the x- and y-axes as shown.

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Mass on the left side should be twice the mass of the right
side, since it has twice the area.
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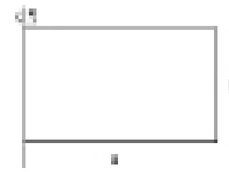
$$\text{left: } \lambda_L = M_L / b = \left(\tfrac{2}{3} M/b\right) \;;\; dm = \lambda_L dx$$
$$\text{right: } \lambda_R = M_R / b = \left(\tfrac{1}{3} M/b\right) \;;\; dm = \lambda_R dx$$
$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_{left} x\,dm + \frac{1}{M}\int_{right} x\,dm$$
$$= \frac{1}{M}\int_0^b x\lambda_L dx + \frac{1}{M}\int_b^{2b} x\lambda_R dx$$
$$= \frac{\lambda_L}{M}\int_0^b x\,dx + \frac{\lambda_R}{M}\int_b^{2b} x\,dx$$
$$= \frac{\lambda_L}{M}\frac{x^2}{2}\Big|_0^b + \frac{\lambda_R}{M}\frac{x^2}{2}\Big|_b^{2b}$$
$$= \frac{\lambda_L}{M}\frac{b^2}{2} + \frac{\lambda_R}{M}\frac{(4b^2 - b^2)}{2}$$
$$= \frac{\left(\tfrac{2}{3} M/b\right)}{M}\frac{b^2}{2} + \frac{\left(\tfrac{1}{3} M/b\right)}{M}\frac{(4b^2 - b^2)}{2}$$
$$= \frac{b}{3} + \frac{b}{2} = \frac{2b}{6} + \frac{3b}{6} = \frac{5b}{6} = 0.8333b$$

OR

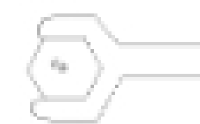
$$A = (4a)(b) + (2a)(b) = 6ab$$
$$\sigma = \frac{M}{A} = \frac{M}{6ab}$$
$$dm = \sigma\,dA = \sigma\,dx\,dy$$
$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int x\,\sigma\,dx\,dy$$
$$= \frac{\sigma}{M}\left[\int_0^b\int_0^{4a} x\,dx\,dy + \int_b^{2b}\int_a^{3a} x\,dx\,dy\right]$$
$$= \frac{\sigma}{M}\left[\int_0^b x\,dx\int_0^{4a} dy + \int_b^{2b} x\,dx\int_a^{3a} dy\right]$$
$$= \frac{\sigma}{M}\left[\frac{x^2}{2}\Big|_0^b (4a) + \frac{x^2}{2}\Big|_b^{2b}(2a)\right]$$
$$= \frac{\sigma}{M}\left[\frac{b^2}{2}(4a) + \frac{4b^2 - b^2}{2}(2a)\right]$$
$$= \frac{\sigma}{M}\left[\frac{b^2}{2}(4a) + \frac{3b^2}{2}(2a)\right]$$
$$= \frac{\sigma}{M}\left[2ab^2 + 3ab^2\right]$$
$$= \frac{\sigma}{M}\left[5ab^2\right]$$
$$= \frac{1}{M}\left(\frac{M}{6ab}\right)\left[5ab^2\right]$$
$$= \frac{5b}{6} = 0.8333b$$



(b) A plate of mass m has sides of a and b. Find the moment of inertia I of the plate about the y-axis as shown, by integrating $I = \int r^2\,dm$.

$$\lambda = \frac{M}{a} \;;\; dm = \lambda dx \;;\; r \to x$$
$$I = \int r^2\,dm = \int_0^a x^2\,\lambda dx = \lambda\int_0^a x^2 dx$$
$$= \lambda\left[\frac{x^3}{3}\right]_0^a = \lambda\left(\frac{a^3}{3}\right)$$
$$= \left(\frac{M/a}{3}\right)(a^3) = \tfrac{1}{3}Ma^2$$

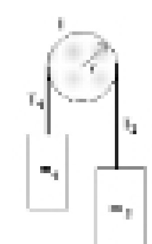
(c) A torque $\vec{\tau}$ to tighten a bolt consists of a force being applied at a distance from the axis of rotation. As the bolt gets tighter, it gets harder and harder to turn the bolt, so the torque as a function of angle is given by $\tau = C\,\theta^2$, where C is some constant with appropriate units. If the total work done by applying this torque through two complete revolutions is 1500. J, then find C.



$$W = \int \tau\,d\theta = \int_0^{4\pi} C\theta^2 d\theta$$
$$= C\frac{\theta^3}{3}\Big|_0^{4\pi} = \frac{C}{3}\left((4\pi\,rad)^3 - 0\right)$$
$$= C\left(661.5\,rad^3\right) = 1500.\,J$$
$$C = \frac{1500.\,J}{\left(661.5\,rad^3\right)} = 2.267\,{}^{J}\!/_{rad^3} \;\; OR \;\; 2.267\,{}^{N\cdot m}\!/_{rad^3}$$

Those quasi-units are slippery!

(d) In the diagram at right, the mass m_1 is moving *down*. Does $\vec{\tau}$ on the pulley point *up, down, left, right, in, out* ?



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Rotation is counter-clockwise, so using RHR, the
thumb points OUT.
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The Torque vector $\vec{\tau}$ points OUT of the page.