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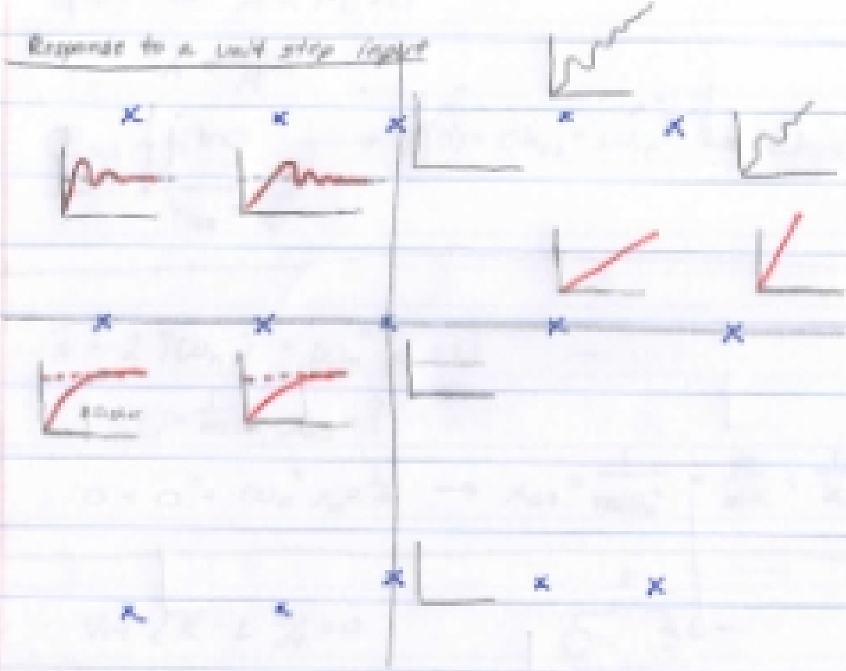
Transfer function Poles
Diff. Equation Roots
State Space Eigenvalues
} define transient response

Response to a unit step input



- Rise time is the response to change in the system

$T_s < 0.5$ sec
% overshoot $< 10\%$
} $\zeta =$
$\omega_n =$

$$T_s \approx \frac{4}{\zeta\omega_n} = .5 \rightarrow \zeta\omega_n = 8$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \rightarrow$$

↳ % overshoot = 4%

↳ $\zeta = .707$

$A\dot{h} + ch = w_{in}$

$w_{in} = 1 \quad h_{ss} = ?$

$\lambda = ? \rightarrow A\lambda + c = 0$

$\lambda = \frac{-c}{A}$

$@_{ss} \rightarrow \dot{h} = 0 \longrightarrow A(\dot{0}) + ch_{ss} = w_{in} \rightarrow ch_{ss} = 1$

$h_{ss} = \frac{1}{c}$

* during steady state all time derivative variables are equal to 0.

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = U$

$U = \frac{1}{m} \quad x_{ss} = ?$

$0 + 0 + \omega_n^2 x_{ss} = \frac{1}{m} \longrightarrow x_{ss} = \frac{1}{m\omega_n^2} = \frac{m}{mk} = \frac{1}{k}$

$V_{in} - iR - L\frac{di}{dt} = 0$

$V_{in} = 1$

$i_{ss}R = 1 \rightarrow i_{ss} = \frac{1}{R}$

$-R - L\lambda = 0 \longrightarrow \lambda = \frac{-R}{L}$

$0 = -k_1x_1 + k_2x_2 - k_2x_1 \longrightarrow \underline{k_2x_1 = k_2x_2 - k_1x_1}$

$0 = -k_2x_2 + k_2x_1 + F$

$0 = -k_2x_2 + k_2x_2 - k_1x_1 + 1$

$\hookrightarrow k_1x_{1ss} = 1 \longrightarrow x_{1ss} = \frac{1}{k_1}$

$a\dot{x} + bx = U$

$\mathcal{L} \rightarrow [as + b]\, Y(s) = U(s)$

$G(s) = \frac{1}{as + b}$

$X_{ss} = ?$ if $U = 1$ $\longrightarrow$ $U(t) =$ unit step $- \mathcal{L}[1] = \frac{1}{s}$

$Y_{ss} = ? \longrightarrow Y(s) = [U(s)][G(s)] = \left(\frac{1}{s}\right)\left(\frac{1}{as + b}\right)$

$Y_{ss} = \lim_{t \to \infty} Y(t) = \lim_{s \to 0} s\, Y(s)$

$Y_{ss} = \lim_{s \to 0} s \left(\frac{1}{s}\right)\left(\frac{1}{as + b}\right) = \frac{1}{b}$

$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = U \qquad U = \frac{F}{m}$

$G(s) = \frac{1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$X_{ss} = \frac{1/m}{\omega_n^2} = \frac{1}{k}$

$m_1 s^2 X_1(s) + k_1 X_1(s) - k_2 X_2(s) + k_2 X_1(s) - b_1 s X_2(s) + b_1 s X_1(s) = 0$

$(m_1 s^2 + k_1 + k_2 + b_1 s) X_1(s) - (k_2 + b_1 s) X_2(s) = 0$

plugging in

$X_2 = \frac{(m_1 s^2 + b_1 s + k_1 + k_2) X_1}{b_1 s + k_2}$

$(m_2 s^2 + b_1 s + k_2] X_2(s) = (b_1 s + k_2) X_1 + F$

$G_1(s) = \frac{X_1}{F} = \frac{b_1 s + k_2}{(m_2 s^2 + b_1 s + k_2)(m_1 s^2 + b_1 s + k_1 + k_2) - (b_1 s + k_2)^2}$

$f(t) = 1 \longrightarrow F(s) = \frac{1}{s}$