1st Ord. Lin. Sys.

Monday, October 26, 2015 10:38 AM

$$\vec{Y}' = P(t) \cdot \vec{Y} + \vec{G}(t)$$

- $P(t)$ is an $n \times n$ matrix
- $\vec{Y} = 1 \times n$,
- $G(t) = 1 \times n$

Ex: Write the system as a first order linear system.

$$y_1' = \sin(t) + \frac{t y_2}{t^2 - 2t - 8} + \ln|t|$$

$$y_2' = (2t+1) + e^{-2t} y_2 + \cos(t)$$

$$y_2' = (2t+1) + e^{-2t} y_2 + \cos(t)$$

$$\vec{y}' = P(t)\,\vec{y} + G(t)$$

$$\vec{y}' = \begin{bmatrix} \sin(t)\, y_1 + \dfrac{t}{t^2-2t+8}\, y_2 \\ (2t+1) y_1 + e^{-2t} y_2 \end{bmatrix} + \begin{bmatrix} \ln(t) \\ \cos(t) \end{bmatrix}$$

$$y' = P(t)\,\vec{y}' + \begin{bmatrix} \ln(t) \\ \cos(t) \end{bmatrix}$$

$$y' = \begin{bmatrix} \sin t & \dfrac{t}{t^2-2t+8} \\ 2t+1 & e^{-2t} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} \ln[t] \\ \cos(t) \end{bmatrix}$$

Ex 2: $$y_1' = \frac{3t\, y_1}{(t+2)} + \frac{5 y_2}{(t+2)}$$

$(t+2)$ $(t+2)$

$$y_2' = \frac{2y_1}{(t-2)} \frac{4ty_2}{(t-2)}$$

$$y' = \begin{bmatrix} \frac{3t}{t+2} & \frac{5}{t+2} \\ \frac{2}{t-2} & \frac{4t}{t-2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} G(t)$$

initial condition: $\vec{y}(-3) = \begin{bmatrix} 5 \\ \sqrt{2} \end{bmatrix}$

$t_0 = -3$

$-\infty < t < -2$ ← largest open interval