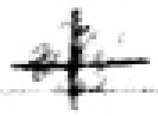
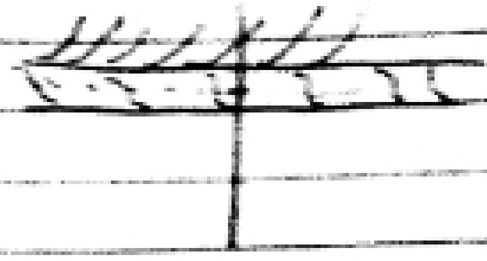
1.1 1-13
1.2 1-3, 7, 8, 9, 15

Slope field 

$\frac{dy}{dx} = y \rightarrow \frac{dy}{y} = 1 \rightarrow \int \frac{dy}{y} = \int dx + C \rightarrow e^{\ln y} = e^{x+C} \quad y = e^{x}e^{C} \rightarrow Ce^{x}$

$\frac{dy}{dx} = y(y-2)$



$F_D = \omega v$

$F_g = 9.8m$

$F = F_g - F_D$

$m\frac{dv}{dt} = 9.8m - \omega v \qquad \frac{dv}{dt} = 9.8 - \frac{\omega v}{m} = \frac{dv}{dt} = 9.8 - \frac{v}{6}$

weight = 90 kg
$\omega = 15$

$0 = 9.8 - \frac{v}{6}$

$y' = 9.8 - \frac{1}{6}v \qquad y' = a - by \qquad 0 = 58.8 = v$

$v' = -\frac{1}{6}(-58.8 + v) \rightarrow \frac{v'}{v - 58.8} = -\frac{1}{6} \rightarrow \left(\ln(v-58.8)\right)' = -\frac{1}{6}$

$\ln(v - 58.8) = \int -\frac{1}{6}\,dT$

$\ln(v - 58.8) = -\frac{1}{6}T + C$

$v - 58.8 = e^{-\frac{1}{6}T + C}$

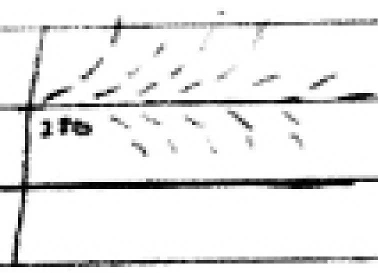
$\boxed{v = Ce^{-\frac{1}{6}T} + 58.8}$

13-14 $\quad dm/dt = rm \qquad$ Throw in 2 cats that kill 5 mice each per day

$r = 0.25$ per week

$dm/dt = rm - 70$

$m' = 0.25m - 70 \qquad 0 = 70/0.25 = m$



$m' = 0.25(250) - 70$

$m = -7.5 \qquad m' = 0.25(m - 280)$

$\left(\ln(m - 280)\right)' = 0.25$

$m = 280 - Ce^{0.25(0)} \qquad \ln(m - 280) = 0.25T + C$

$250 = \qquad m - 280 = Ce^{0.25T}$

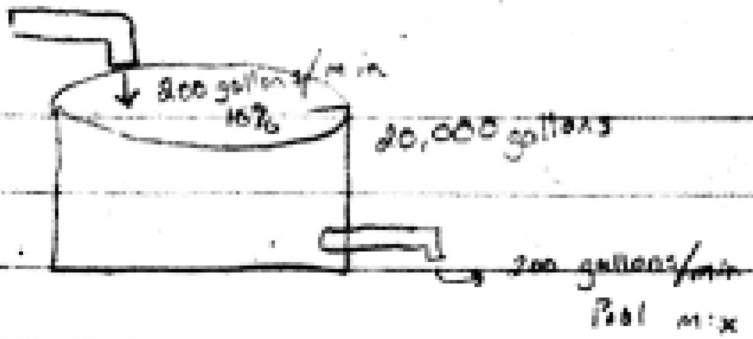
$C = 30 \qquad m = Ce^{0.25T} + 280$

$0 = 280 - 30e^{0.25T} \qquad \boxed{m = 280 - 30e^{1/4 T}}$

$9.3 = e^{0.25T}$

$2.23 = 0.25T$

$8.93 = T$



Write a differential equation for a $Q(T)$ which is the amount of chlorine in the tank after T min

$Q'(T) = 0.10(200\ \text{gal/min}) - \frac{Q(200\ \text{gal/min})}{20{,}000}$

$Q'(T) = 20 - Q(0.01)$

$0 = 20 - 0.01Q \qquad Q = 2000$ gallons

$Q_0(1200)$

$Q'(t) = 0.01(2000 - Q)$

$(\ln(2000-Q))' = 0.01 \qquad 2000 - Q = Ce^{0.01T} \qquad Q(T) = 2000 - Ce^{0.01T}$

$12000 - 2000 = -C$

$-10000 = C$

$Q'(T) = 2000 + 10000e^{0.01T}$

$\frac{-2000}{10000} = e^{0.01T}$

A bowling Ball with a drag coefficient of 0.8 is dropped from a 900 ft Building
find $\frac{dv}{dt}$ $\quad g = 9.8\ m/s^2 \quad 5\text{kg} \qquad F_w = kv$

$F = F_g - F_w \qquad 5\text{kg}(9.8\ m/s^2) - (0.8)\,dv/dt$

$m\left(\frac{dv}{dt}\right) = 49 - 0.8v' \qquad v' = 9.8\ m/s^2 - \frac{0.8}{5}\,dv/dt$

$\ln(61.25 - v)$

5/14/14

$3y' + (T+4)y = T^2 + y''$ linear $\qquad y''' = \cos(2Ty)$ Not linear

$3y' + Ty + 4y = T^2 + y''$ $\qquad$ when y is embedded in the function then it's Non linear

Not linear

linear or Non-linear $\qquad f(x,y) = 3xy + 2x^2y$

ordinary or Partial $\qquad \frac{df}{dx} = 3y + 4yx$

will get to these:
- Seperable $\qquad \frac{df}{dy} = 3x + 2x^2$
- exact
- Autonomous
- Homogeneous $\qquad$ Things, Non-linear $(\text{Trig}(y),\ e^y,\ y^2)$

$\underbrace{(x^2+3)y' + (2x)y}_{T} = x \qquad y' + P(x)y = g(x)$

$\int((x^2+3)y)' = \int x \qquad (x^2+3)y = \frac{x^2}{2} + \qquad \boxed{y = \frac{x^2}{2x^2+6} + \frac{C}{x^2+3}}$

$e^{3x} \quad y' + 3y = x$ $\qquad$ integration by parts $\int u\,dv = uv - \int v\,du$

$y'e^{3x} + 3e^{3x}y = xe^{3x} \qquad u = x \rightarrow du = dx \qquad e^{3x}y = \frac{xe^{3x}}{3} - \int\frac{e^{3x}}{3}$

$\int(e^{3x}y)' = \int xe^{3x} \qquad dv = e^{3x} \rightarrow v = \frac{e^{3x}}{3}$

$\frac{xe^{3x}}{3} - \frac{e^{3x}}{9}$

$e^{3x}y =$

$y = \frac{x}{3} - \frac{1}{9} + \frac{C}{e^{3x}}$

$\frac{1}{3} - \cancel{3}e^{-3x} + 3\left(\frac{x}{3} - \cancel{x} + \frac{c}{e^{3x}}\right)$

$\frac{1}{3} - 3ce^{3x} + x - \frac{1}{3} + \frac{3c}{e^{3x}} = x$

$y' + P(x)y = g(x)$

$\mu(x)y' + \mu(x)P(x)y = \mu(x)g(x)$

$\mu(x)y' + \mu(x)P(x)y = (\mu(x)y)'$

$\mu(x)y' + \mu(x)P(x)y = \mu(x)'y + \mu(x)y'$

$\mu(x)P(x)y = \mu(x)'y$

$\mu(x)P(x) = \mu'(x)$

$P(x) = \frac{\mu'(x)}{\mu(x)}$

$P(x) = (\ln(\mu(x)))'$ $\quad \int P(x) = \ln(\mu(x))$

$e^{\int P(x)} = \mu(x)$

Example $\quad y' = 2y + 4$

old way $\quad y' = 2(y+2)$

$\rightarrow \int \frac{y'}{(y+2)} = \int 2 \qquad \ln(y+2) = 2x + C$

$y + 2 = Ce^{2x}$

$\boxed{y = Ce^{2x} - 2}$

New way integrating factors $\quad y' - 2y = 4$

$\mu(x) = e^{\int -2} = e^{-2x}$

$e^{-2x}y' - 2e^{-2x}y = 4e^{-2x}$

$\int (e^{-2x}y)' = \int 4e^{-2x}$

$e^{-2x}y = -2e^{-2x} + C$

$y = -2 + \frac{C}{e^{-2x}} \rightarrow \boxed{y = -2 + Ce^{2x}}$

---

25 $\quad du/dt = -k(u-T)$

$\int \frac{du}{(u-T)} = \int -k\,dt \qquad \ln(u-T) = -kt + C$

$u - T = Ce^{-kt} \qquad u = Ce^{-kt} + T$

a) $\quad u = (u_0 - T)e^{-kt} + T \qquad u_0 = T + Ce^0$

B) $\quad (u_0 - T)e^{-kt} = \frac{1}{2}(u_0 - T) \qquad u_0 - T = C$

$e^{-kt} = \frac{1}{2} \qquad -kt = \ln(\frac{1}{2})$

$kt = -\ln(\frac{1}{2})$

$y' + 2y = x$

$e^{2x} \quad e^{2x}y' + 2e^{2x}y = xe^{2x} \qquad \int (e^{2x}y)'dx = \int xe^{2x}dx \qquad \int xe^{2x} =$

$e^{2x}y = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} - C \qquad u = x \quad du = 1$

$dv = e^{2x} \quad v = \frac{e^{2x}}{2}$

$y = \frac{x}{2} - \frac{1}{4} - \frac{2}{e^{2x}} \qquad y = \frac{x}{2} - \frac{1}{4} - \frac{c}{e^{2x}} \qquad \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}$

$\frac{7}{4} = \frac{x}{2} - \frac{1}{4} + \frac{c}{e^{2x}} \qquad \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$

$2 = \frac{x}{2} - \frac{c}{e^{2x}}$

$2 = -C$

$y' - 3y = -6x - 1 \qquad y(0) = 1$

$e^{3x}y' - 3e^{3x}y = -1(6x+1)e^{3x} \qquad u = -6x - 1 \quad du = -6 \qquad 1 = \frac{1}{3} + \frac{2}{3} + C$

$\int (e^{3x}y)' = \int -1(6x+1)e^{3x} \qquad dv = e^{3x} \quad v = \frac{-e^{3x}}{3} \qquad C = 0$

$e^{3x}y = \frac{(-6x-1)e^{3x}}{3} - \int -2e^{3x}$

$e^{3x}y = \frac{-6x-1}{-3}e^{-3x} + \frac{2e^{-3x}}{3} - C \qquad y = 2x + \frac{1}{3} + \frac{2}{3} + \frac{C}{e^{3x}}$

$\boxed{y = 2x + 1}$