Math 19: Calculus **Winter 2008** **Instructor: Jennifer Kloke**

Practice Final

1. Determine whether each statement is true or false. Unless otherwise stated, any function below is arbitrary. **If the statement is true, very briefly cite your reasoning. If it is false, provide an example showing the statement to be false.**

   (a) True or False. If $f'(x) < 0$ for $1 < x < 6$, then $f(x)$ is decreasing on (1,6).

   Solution: True. The $f'(x) < 0$ if and only if $f(x)$ is decreasing. $\square$

   (b) True or False. If $f(x)$ has an absolute minimum value at $x = c$, the $f'(c) = 0$.

   Solution: False. $f(x) = |x|$ has absolute minimum at $x = 0$ but $f'(0)$ does not exist. $\square$

   (c) True or False. $f'(x)$ has the same domain as $f(x)$.

   Solution: False. $f(x) = |x|$ is defined everywhere but $f'(x)$ is not defined when $x = 0$. $\square$

   (d) True or False. If $f(x)$ and $g(x)$ are differentiable, then

   $$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

   Solution: True. Chain rule. $\square$

   (e) True or False. A function can have two different horizontal asymptotes.

   Solution: True. $\lim_{x\to-\infty} f(x)$ does not have to equal $\lim_{x\to\infty} f(x)$ such as when $f(x) = \arctan x$. $\square$

   (f) True or False. $\lim_{x\to 4}\left(\frac{2x}{x-4} - \frac{8}{x-4}\right) = \lim_{x\to 4}\frac{2x}{x-4} - \lim_{x\to 4}\frac{8}{x-4}$.

   Solution: False. The limit of a difference is the difference of the limits only if both limits exist. $\square$

2. Complete the following sentence:
The function $f(x)$ is continuous on the interval $[a, b]$ if

Solution: $\lim_{x\to c} f(x) = f(c)$ for every number c in the interval (a, b). □

3. Compute the following limits. Justify your results. If the limit is positive or negative infinity, it should be clearly indicated instead of just saying the limit does not exist

(a) $\lim\limits_{x\to\pi} \cos(x + \sin(x))$

Solution:

$$\begin{aligned}\lim_{x\to\pi} \cos(x+\sin(x)) &= \cos(\pi + \sin(\pi)) \text{ b/c continous at } x = \pi. \\ &= \cos(\pi) = 1\end{aligned}$$

□

(b) $\lim\limits_{x\to 1} \dfrac{2-x}{(x-1)^2}$

Solution:

$$\lim_{x\to 1} \frac{2-x}{(x-1)^2} = \infty$$

because the numerator, $2 - x$ approaches 1 as $x \to 1$ while the denominator becomes a very small positive number as $x \to 1$. Thus the fraction becomes a very large positive number as $x \to 1$. □

(c) $\lim\limits_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

Solution:

$$\begin{aligned}\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} &= \lim_{x\to -4} \frac{\frac{x+4}{4x}}{\frac{4+x}{1}} \\ &= \lim_{x\to -4} \frac{1}{4x} \text{ b/c limit doesn't see } x = -4 \\ &= \frac{-1}{16} \text{ b/c continuous at } x = -4.\end{aligned}$$

□

(d) $\lim_{x\to-\infty} \frac{2x^2-5x-2}{x^4-10x^2-3}$

Solution:

$$\begin{aligned}
\lim_{x\to-\infty} \frac{2x^2-5x-2}{x^4-10x^2-3} &= \lim_{x\to-\infty} \frac{\frac{2x^2}{x^4}-\frac{5x}{x^4}-\frac{2}{x^4}}{\frac{x^4}{x^4}-\frac{10x^2}{x^4}-\frac{3}{x^4}} \\
&= \lim_{x\to-\infty} \frac{\frac{2}{x^2}-\frac{5}{x^3}-\frac{2}{x^4}}{1-\frac{10}{x^2}-\frac{3}{x^4}} \\
&= \frac{\lim_{x\to-\infty}\frac{2}{x^2}-\lim_{x\to-\infty}\frac{5}{x^3}-\lim_{x\to-\infty}\frac{2}{x^4}}{1-\lim_{x\to-\infty}\frac{10}{x^2}-\lim_{x\to-\infty}\frac{3}{x^4}} \\
&= \frac{0}{1}=0
\end{aligned}$$

□

4. Use the limit definition of the derivative to compute the derivative of $f(x)=\sqrt{1+2x}$.

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to0}\frac{\sqrt{1+2(x+h)}-\sqrt{1+2x}}{h} \\
&= \lim_{h\to0}\frac{(\sqrt{1+2(x+h)}-\sqrt{1+2x})}{h}\frac{(\sqrt{1+2(x+h)}+\sqrt{1+2x})}{(\sqrt{1+2(x+h)}+\sqrt{1+2x})} \\
&= \lim_{h\to0}\frac{1+2(x+h)-(1+2x)}{h(\sqrt{1+2(x+h)}+\sqrt{1+2x})} \\
&= \lim_{h\to0}\frac{2h}{h(\sqrt{1+2(x+h)}+\sqrt{1+2x})} \\
&= \lim_{h\to0}\frac{2}{(\sqrt{1+2(x+h)}+\sqrt{1+2x})} \text{ b/c limit doesn't see } h=0 \\
&= \frac{2}{(\sqrt{1+2(x+0)}+\sqrt{1+2x})} \text{ b/c continuous at } h=0 \\
&= \frac{1}{\sqrt{1+2x}}
\end{aligned}$$

□