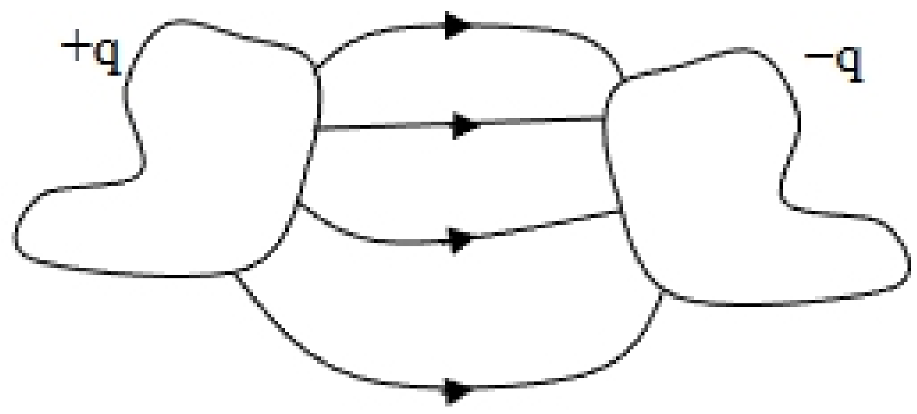
Capacitance

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.



A set of conductors can store electric charge. The net charge Q=0=q−q, but the magnitude of charge on each conductor is $|q|$.

This charge q is proportional to the potential difference between the conductors:

$$q = C\Delta V \qquad \Delta V = V_+ - V_- \equiv V$$
$$q = CV$$

The constant of proportionality between charge and potential difference is C≡capacitance. Unit is Farad (F) ≡ Coulomb/Volt.

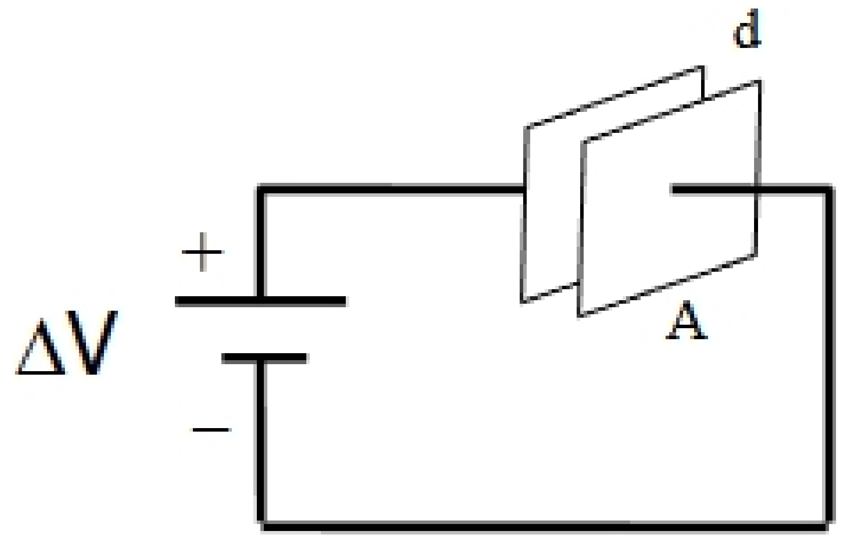
$$1\mu\text{F} = 10^{-6}\ \text{F}$$
$$1\text{pF} = 10^{-12}\ \text{F}$$

To set up a potential difference between 2 conductors requires an electric "pump", such as a battery (see next chapter).

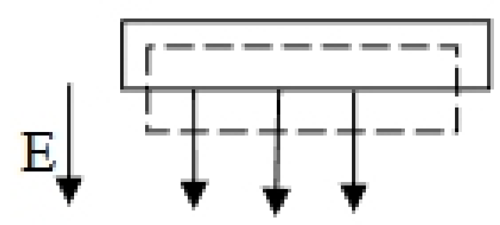
A larger capacitance implies that a large charge q is stored for the same potential difference V.

Capacitance depends only on the geometry of the conductors, not the charge q or voltage V. We can see this through examples.

Parallel Plate Capacitor



Consider the top view of the 2 plates:



Create a Gaussian surface (box) that extends inside and outside one of the conductor surfaces.

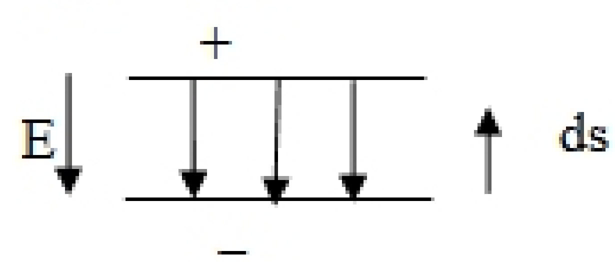
Gauss' Law $\Rightarrow \Phi = \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$

$\mathbf{E} = 0$ inside a conductor
$\mathbf{E} \cdot d\mathbf{A} = 0$ on left/right edges
$|\mathbf{E}| \neq 0$ on front outside face only

$\Rightarrow \Phi = \oint_S \mathbf{E} \cdot d\mathbf{A} = EA = \frac{q_{\text{enc}}}{\varepsilon_0}$

$\Rightarrow q = \varepsilon_0 EA$

The electric potential difference between the 2 plates is given by:



$\Delta V = V_+ - V_- = -\int_-^+ \mathbf{E} \cdot d\mathbf{s}$ $\quad \mathbf{E} \cdot d\mathbf{s} = -|\mathbf{E}|\, ds$ opposite directions

$\Delta V = |\mathbf{E}|\, d$

$\Rightarrow |\mathbf{E}| = \frac{\Delta V}{d}$

So for parallel plates:

$q = \varepsilon_0 EA = \varepsilon_0 \frac{\Delta V}{d} A = \left(\varepsilon_0 \frac{A}{d} \right) \Delta V$

$q = C \Delta V$

$\Rightarrow \boxed{C = \varepsilon_0 \frac{A}{d}}$ $\quad \varepsilon_0 = 8.85 \times 10^{-12} \ \frac{\text{C}^2}{\text{Nm}^2} = 8.85 \text{ pF/m}$

Cylindrical Capacitor (Cable)

Let inner conductor have radius a, and outer radius b.
Take Gaussian surface as cylinder between conductors (**E**=0 inside conductors).

$$\Phi = \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E 2\pi r L \varepsilon_0 = q \qquad \mathbf{E} \cdot d\mathbf{A} = 0 \text{ on cylinder ends}$$

$$\Rightarrow E = \frac{q}{2\pi\varepsilon_0 L}\frac{1}{r}$$

$$\Delta V = V_+ - V_- = -\int_-^+ \mathbf{E} \cdot ds \qquad \mathbf{E} \cdot ds = -|\mathbf{E}|\, ds \quad \text{opposite directions, but } ds = -dr \text{ opposite again}$$

$$\Delta V = -\int_-^+ |\mathbf{E}|\, dr = \int_a^b |\mathbf{E}|\, dr = \int_a^b \frac{q}{2\pi\varepsilon_0 L}\frac{dr}{r}$$

$$\Delta V = \frac{q}{2\pi\varepsilon_0 L}\ln\frac{b}{a}$$

$$\Rightarrow q = \left(\frac{2\pi\varepsilon_0 L}{\ln b/a}\right)\Delta V$$

$$\Rightarrow \boxed{C = \frac{2\pi\varepsilon_0 L}{\ln b/a}}$$

Spherical Capacitor

Let inner sphere have radius a, and outer radius b.
Take Gaussian surface as sphere between conductors (**E**=0 inside conductors).

$$\text{Gauss' Law} \Rightarrow |\mathbf{E}| = K\frac{q}{r^2} \qquad a < r < b$$

$$\Delta V = V_+ - V_- = -\int_-^+ \mathbf{E} \cdot ds \qquad \mathbf{E} \cdot ds = -|\mathbf{E}|\, ds \quad \text{opposite directions, but } ds = -dr \text{ opposite again}$$

$$\Delta V = -\int_-^+ |\mathbf{E}|\, dr = \int_a^b |\mathbf{E}|\, dr = \int_a^b Kq\frac{dr}{r^2}$$

$$\Delta V = Kq\left(-\frac{1}{r}\right)\Bigg|_a^b = Kq\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Rightarrow q = \left(\frac{1}{K}\frac{ab}{b-a}\right)\Delta V$$

$$\Rightarrow \boxed{C = \frac{1}{K}\frac{ab}{b-a}}$$